Approaching the Conformal Window on the Lattice

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arXiv:0910.2224, 1009.5967

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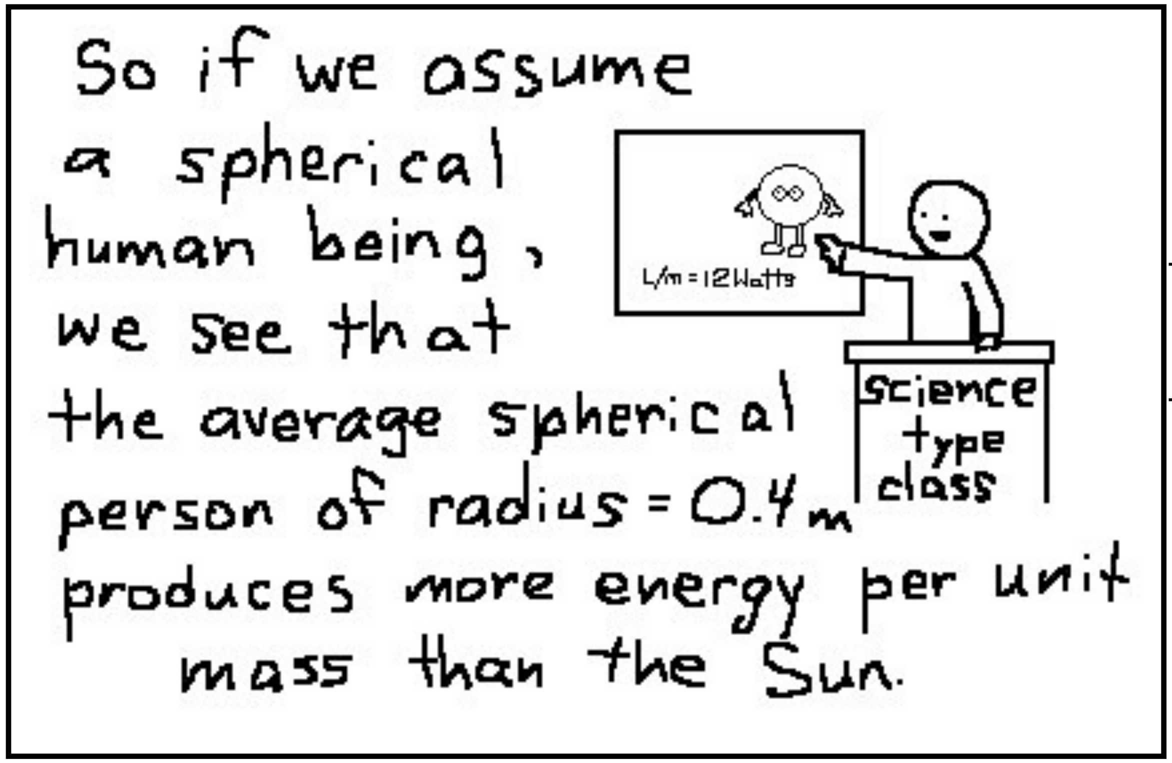
Motivation

- What if BSM physics is strongly coupled?
    - Perturbation theory only goes so far...use assumptions based on QCD phenomenology?
    - Focus on Yang-Mills gauge theories, which can look very different from QCD (e.g. the conformal window - with many fermions, no confinement, no spontaneous xSB)



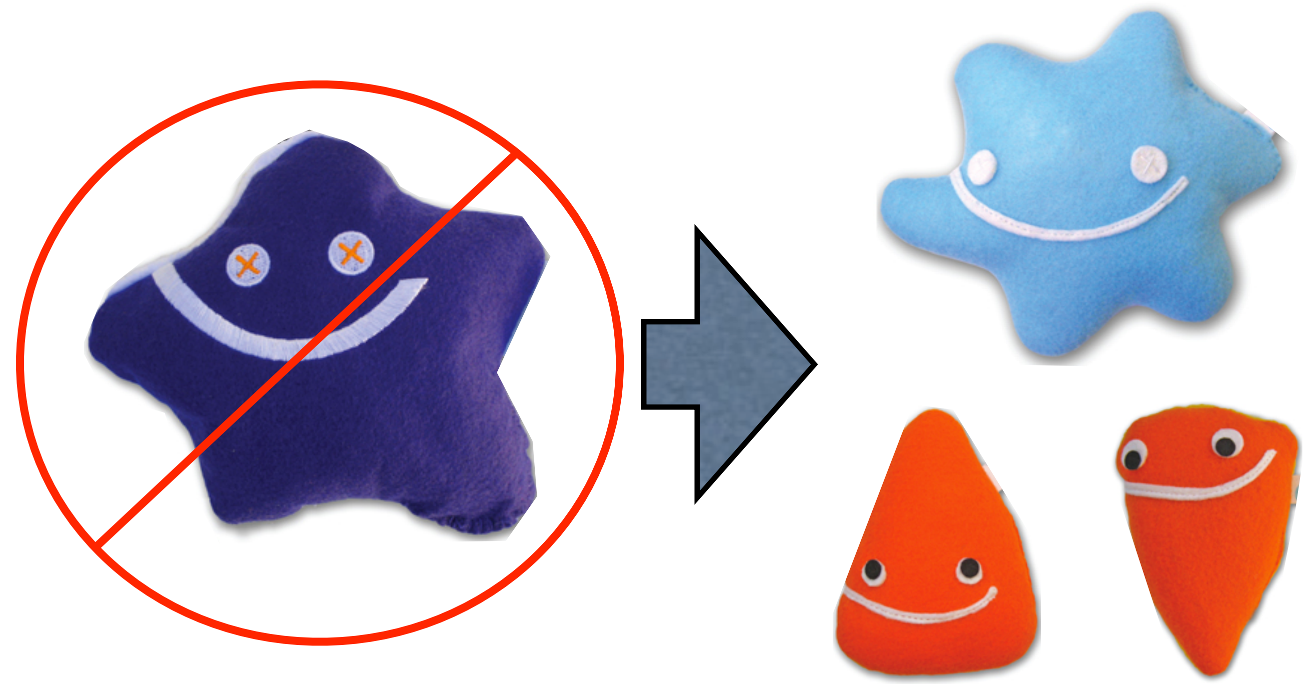
http://www.lapsura.com/

Lattice gauge theory lets us explore strongly-coupled field theories non-perturbatively.

Technicolor, briefly

- Technicolor theories replace the Higgs scalar field with new strong dynamics. Chiral symmetry breaking also breaks electroweak symmetry.
- Typically, new gauge group is SU(N_{TC}), with N_{TF} new Dirac "technifermions".
- Minimal or one-doublet technicolor is QCD, rescaled: $N_{TC}=3$, $N_{TF}=2$.

$$\Lambda_{QCD} \sim 1\ \mathrm{GeV} \rightarrow \Lambda_{TC} \sim 1\ \mathrm{TeV}.$$



http://particlezoo.net

- What about other SM masses?

$$H\overline{\psi}\psi \longrightarrow \overline{T}T\overline{\psi}\psi$$

Effective four-fermion operator from new gauge interactions (extended technicolor.)

S. Dimopoulos and L. Susskind, Nucl. Phys. B155 (1979);
E. Eichten and K. D. Lane, Phys. Lett. B90 (1980)

The trouble with (minimal) technicolor

- First problem: reproducing CKM mixing leads to flavor-changing neutral current (FCNC) operators. Strong constraints from precision electroweak!

- For example, from kaon mixing:

K. D. Lane, hep-ph/0007304

$$(\Delta M_K) < 3.5 \times 10^{-18}\ \text{TeV} \rightarrow \Lambda_2 > 1300\ \text{TeV}$$

second-generation ETC breaking scale
(suppresses four-fermi operators.)

Reduces FCNC contributions and standard model masses!

$(\overline{\psi}\psi\overline{\psi}\psi)$ $(\overline{T}T\overline{\psi}\psi)$

Big problem, if the condensate $\langle \overline{T}T \rangle = \eta \Lambda_{TC}^3$ (where $\eta = \mathcal{O}(1)$).

True in QCD, but in general?

The trouble with (minimal) technicolor

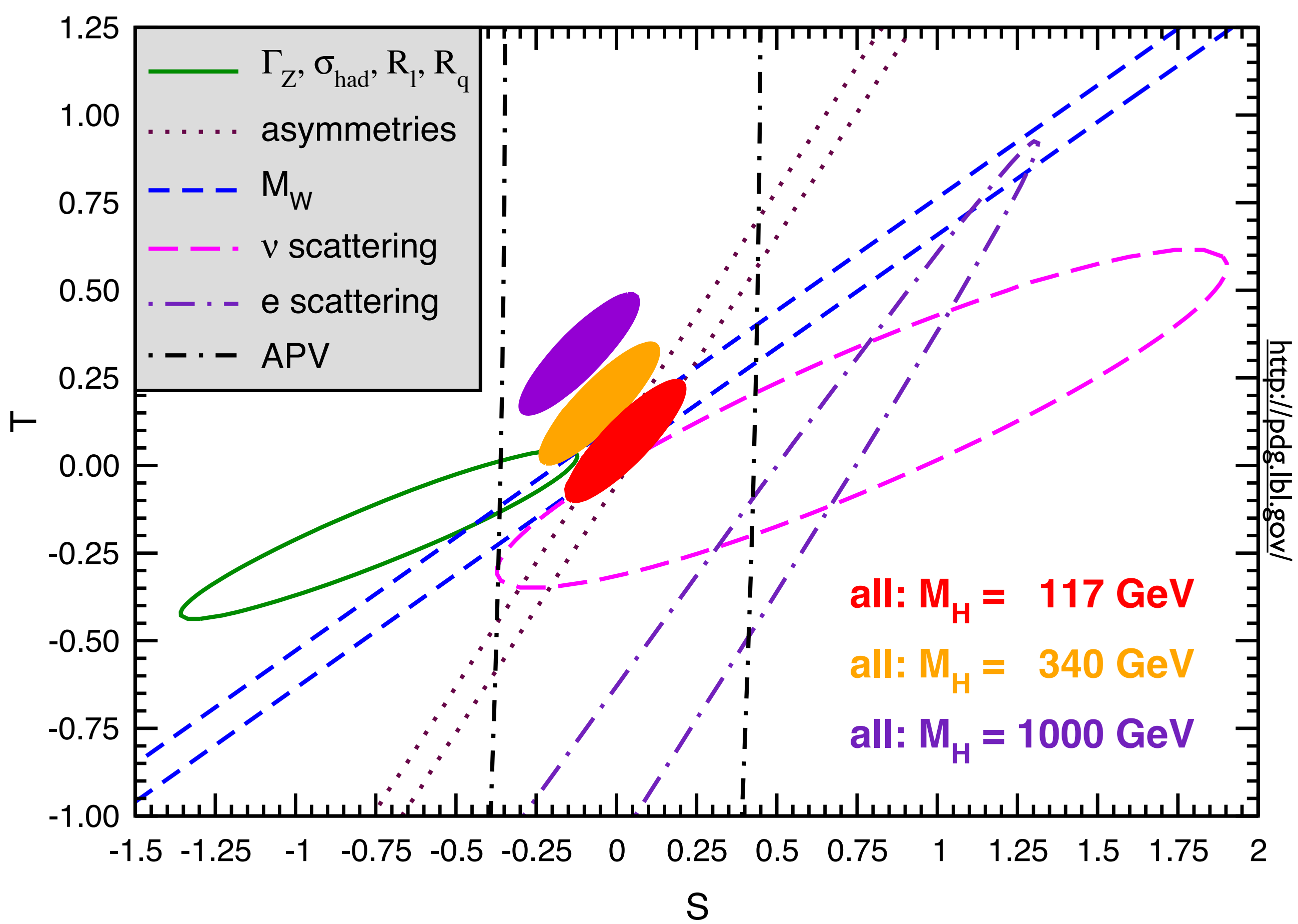
Second problem: if we add particle content, we run into S

S parameter: sensitive to new electroweak physics

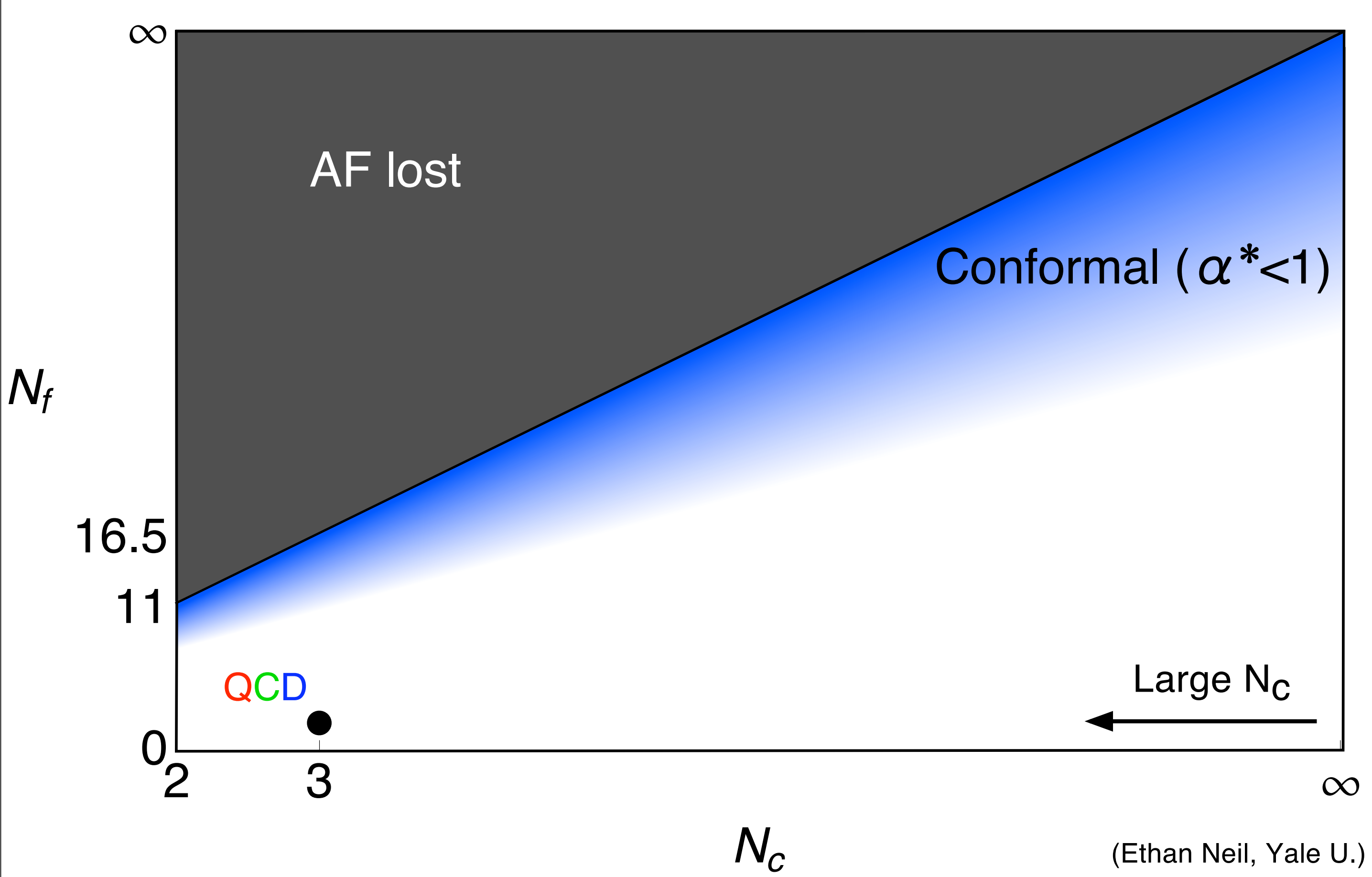
In a technicolor theory (Peskin & Takeuchi '92):

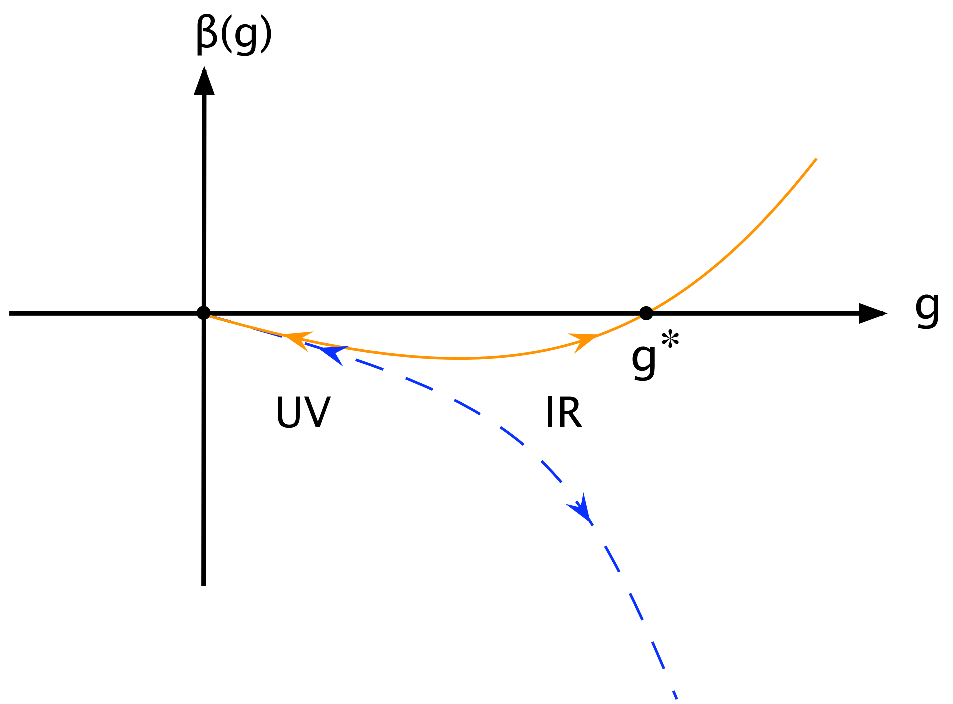
$$S \simeq 0.25 \frac{N_{TF}}{2} \frac{N_{TC}}{3} + \frac{1}{12\pi} \left(\frac{N_{TF}^2}{4} - 1 \right) \log \left(\frac{m_{\rho_T}^2}{m_{\pi_T}^2} \right)$$

relies on QCD pheno!



The Conformal Window

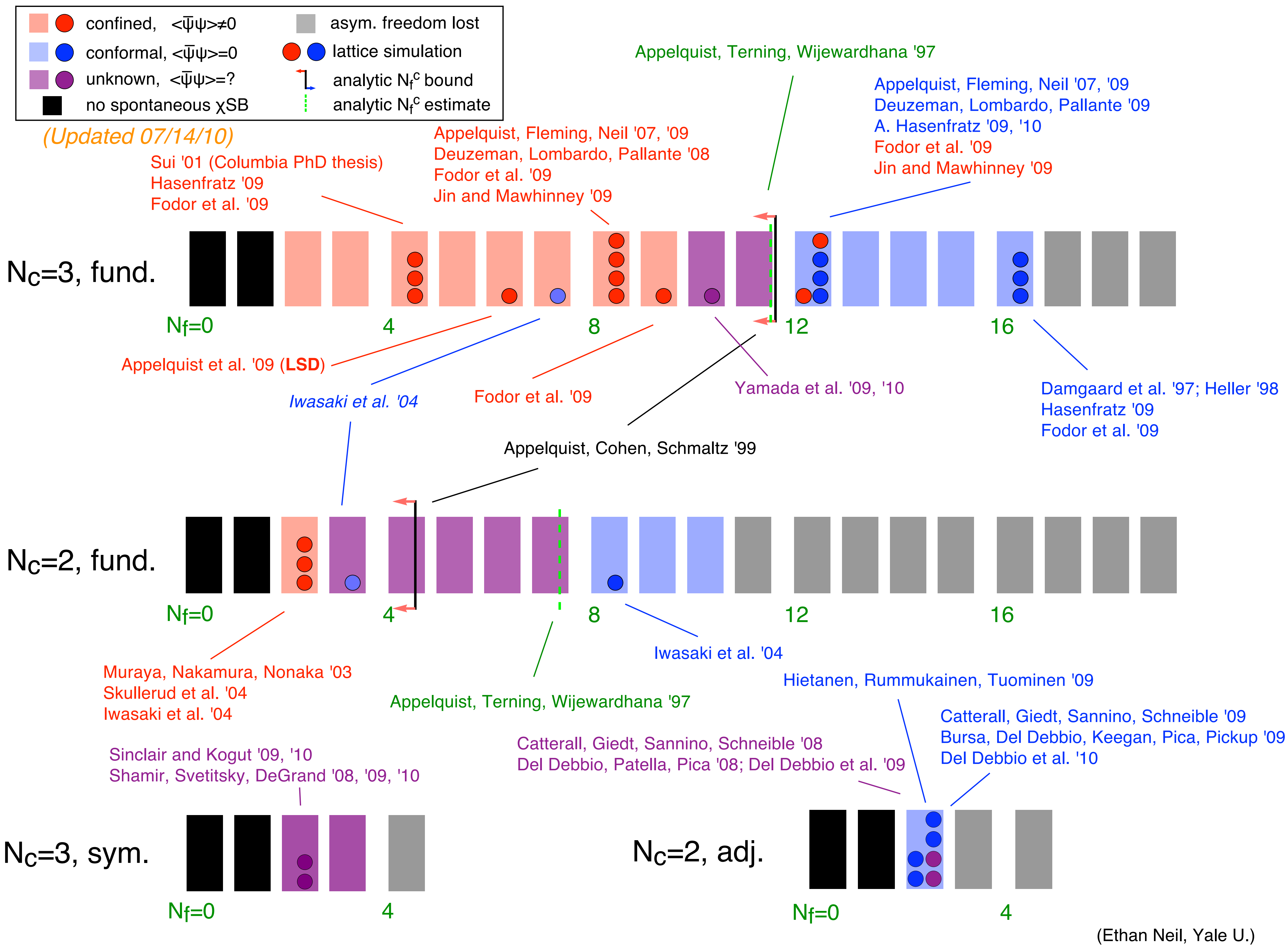




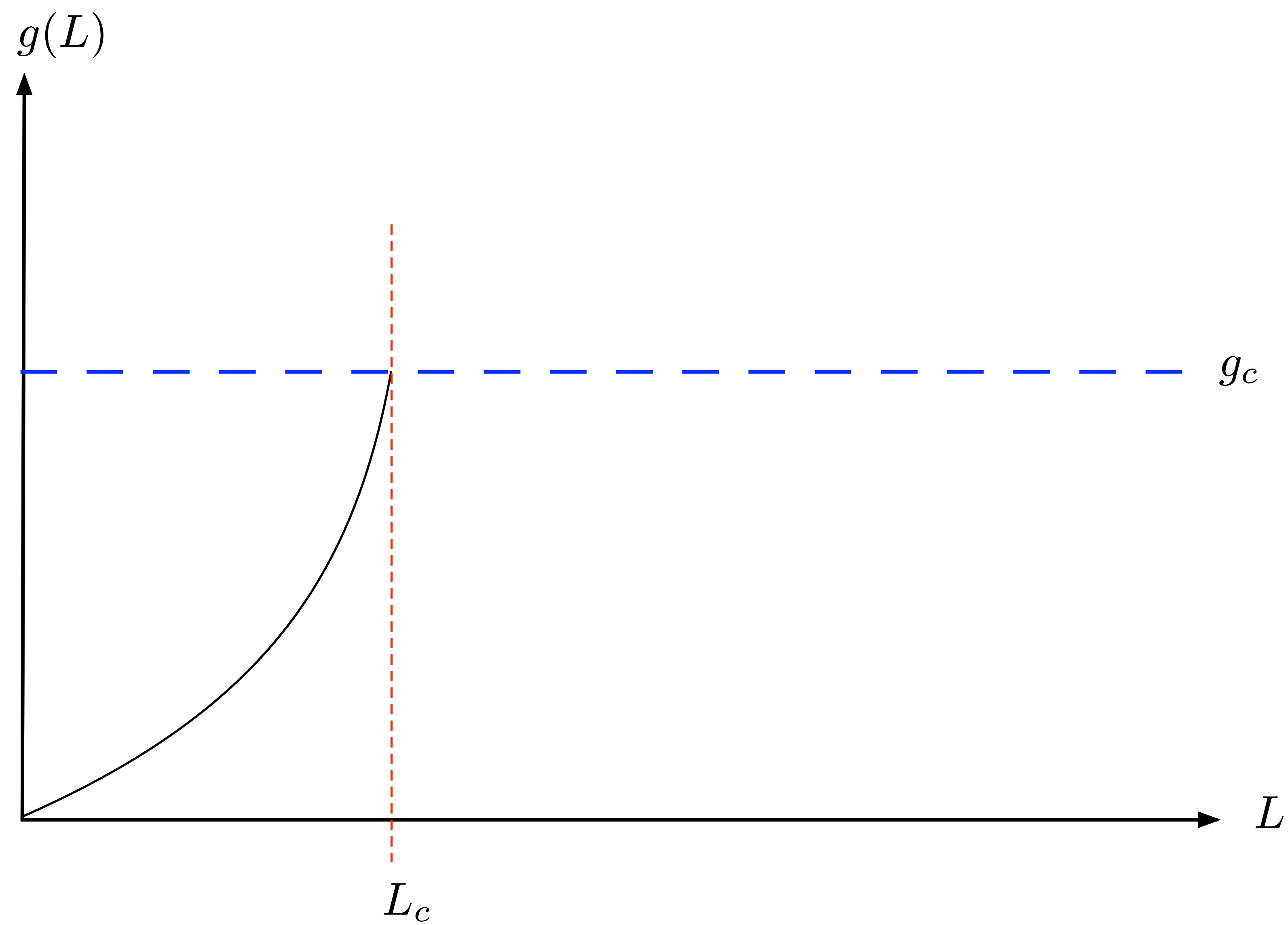
Large N_c expansion works well for QCD, but for large N_f, things change drastically (IR fixed point)

W. Caswell, Phys. Rev. Lett. 33:244,1974
T. Banks and A. Zaks, Ncul. Phys. B 196:189, 1982

A conformal window roadmap



Dynamical scales

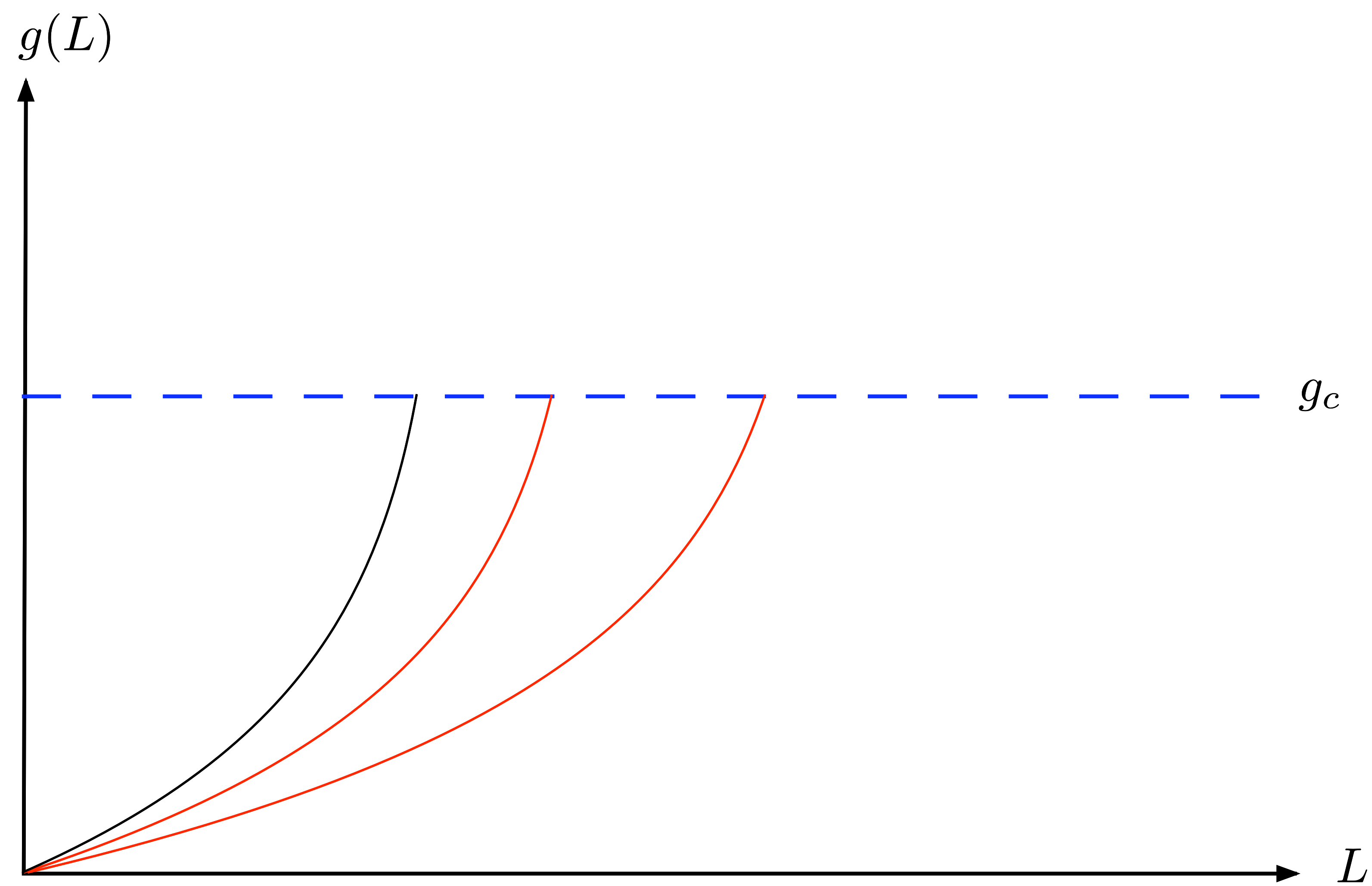


L_c: confinement scale
L_i: inflection-point scale

In a theory with both scales, condensates are enhanced by modes between L_i and L_c!

Appelquist, Terning, Wijewardhana, Phys. Rev. D 44, 871 (1991)

Dynamical scales



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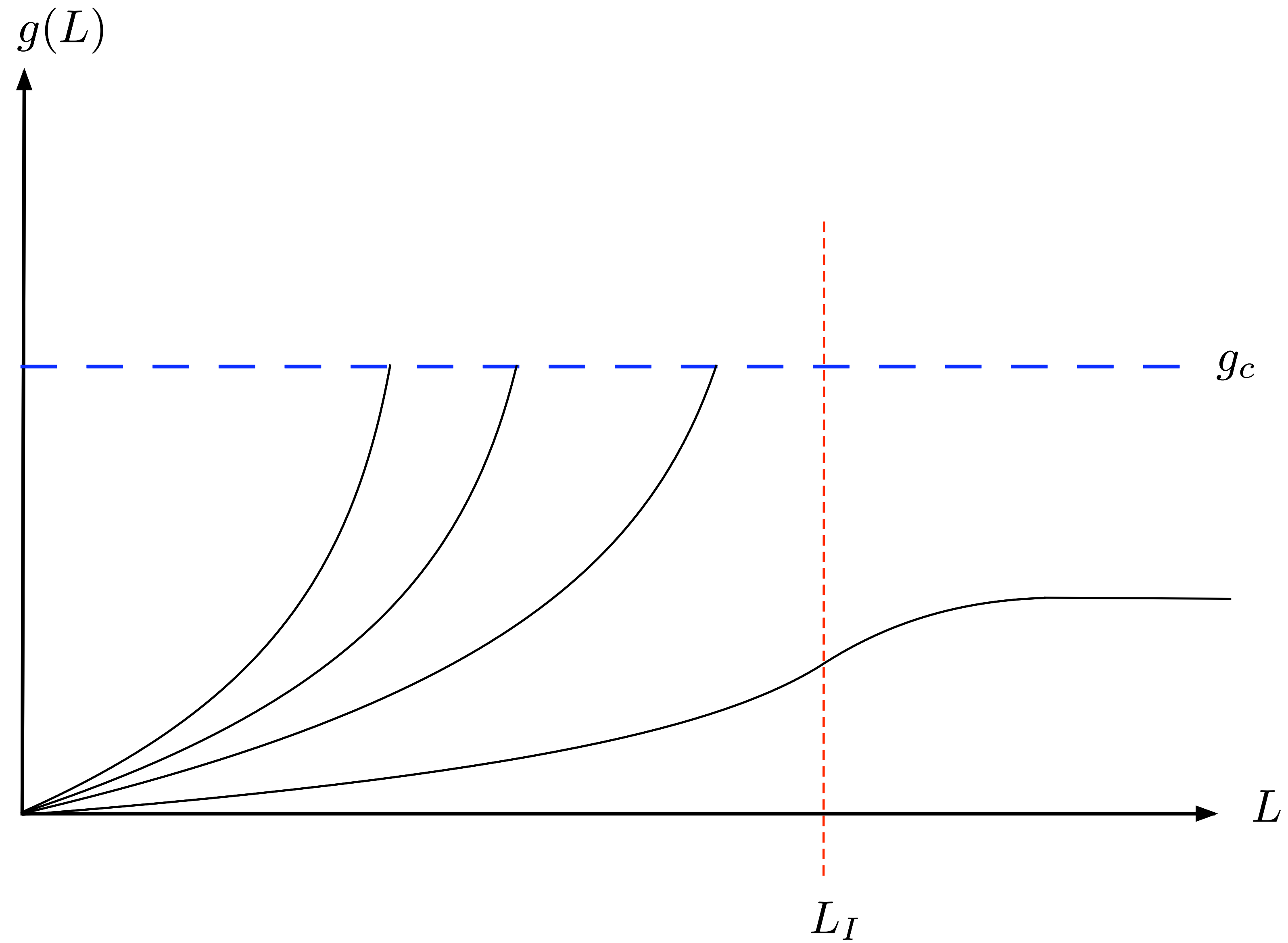

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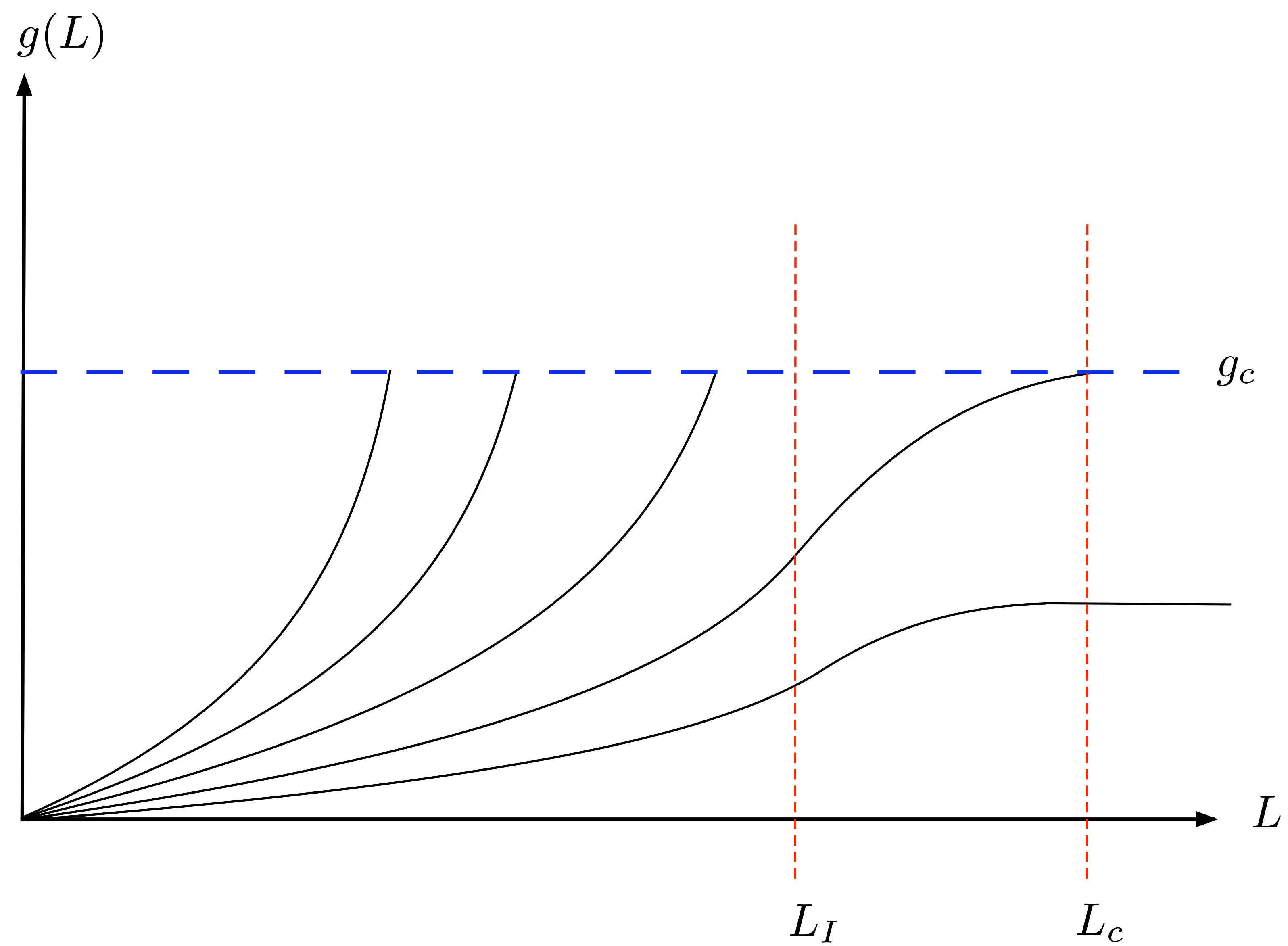

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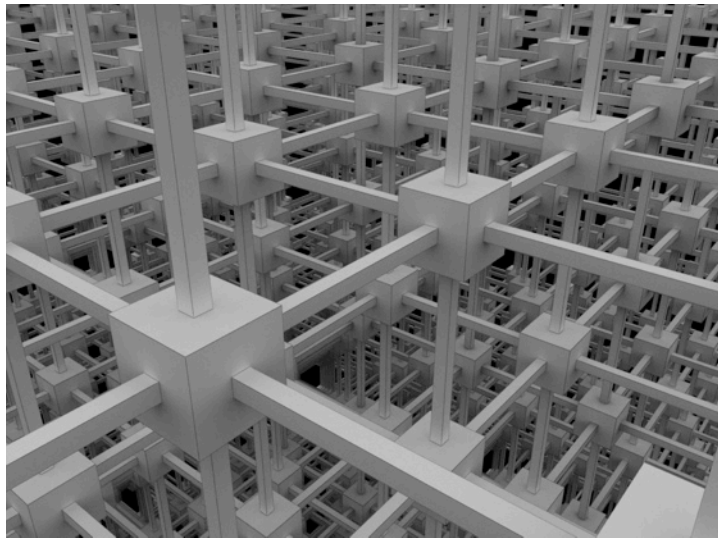
Going to the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \mathcal{D}\overline{\psi} \mathcal{D}\psi \; \mathcal{O}(U, \overline{\psi}, \psi) \exp\left(-S[U, \overline{\psi}, \psi]\right)$$



$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{U \in \mathcal{U}} \langle \mathcal{O} \rangle_U$$

Discretize to make the path integral finite-dimensional (but sharply peaked!)

Importance sampling and Monte Carlo techniques give us an ensemble of field configurations, weighted by exp(-S)

Generating the weighted ensemble is typically the hard part...

Our Strategy

- Approach the conformal window from below, measure various quantities, look for trends as N_f varies.
- This talk: 3 colors, N_f=2 vs. N_f=6.
- Basic measurements:
  - low-lying spectrum and decay constants
  - chiral condensate
  - *S*-parameter
  - (your favorite observable here)

Simulation Details

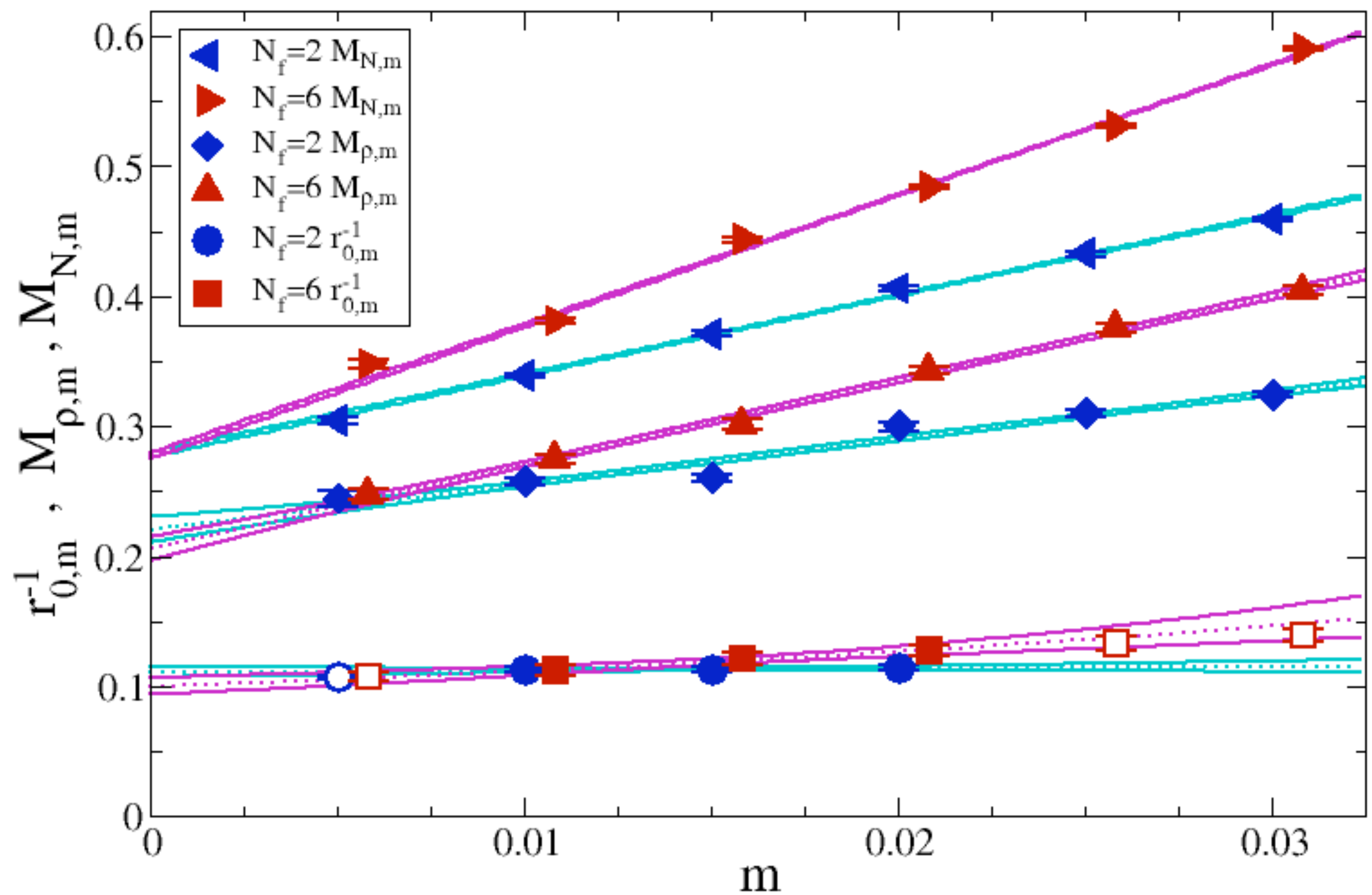
- We use domain wall fermions to preserve as much chiral, flavor symmetry as possible. Residual χSB is small:

$$m_{res} = \begin{cases} 2.6 \times 10^{-5}, & N_f = 2 \\ 8.2 \times 10^{-4}, & N_f = 6 \end{cases}$$

- All volumes are 32^3x64, lattice spacing tuned to $a \sim 5m_\rho$. At 2-flavors, this gives $a \sim 0.06 \text{ fm} = 3.6 \text{ GeV}^{-1}, L \sim 1.8 \text{ fm}$.

	$N_f = 2$		$N_f = 6$	
am_f	"M_π" L	N_{cfg}	"M_π" L	N_{cfg}
0.005	3.5	1430	4.7	1350
0.010	4.4	2750	5.4	1250
0.015	5.3	1060	6.6	550
0.020	6.5	720	7.8	400
0.025	7.0	600	8.8	420
0.030	7.8	400	9.8	360

Setting the scale



To compare theories, fix a physical scale (pick your favorite.)

In general, scales may diverge! Choose from context, e.g. fix decay constant *F ~ v/2* for technicolor.

NLO χPT, general N_f

$$M_m^2/m = 2B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_m + \frac{1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

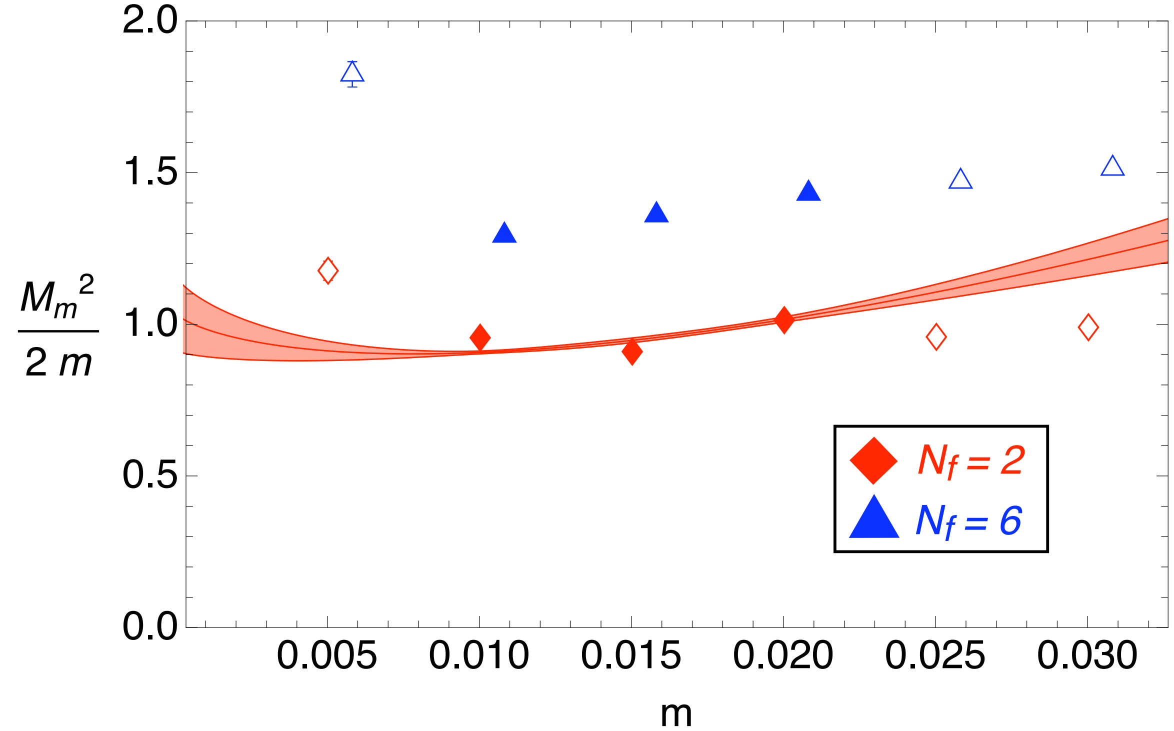
$$F_m = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_F - \frac{N_f}{2}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

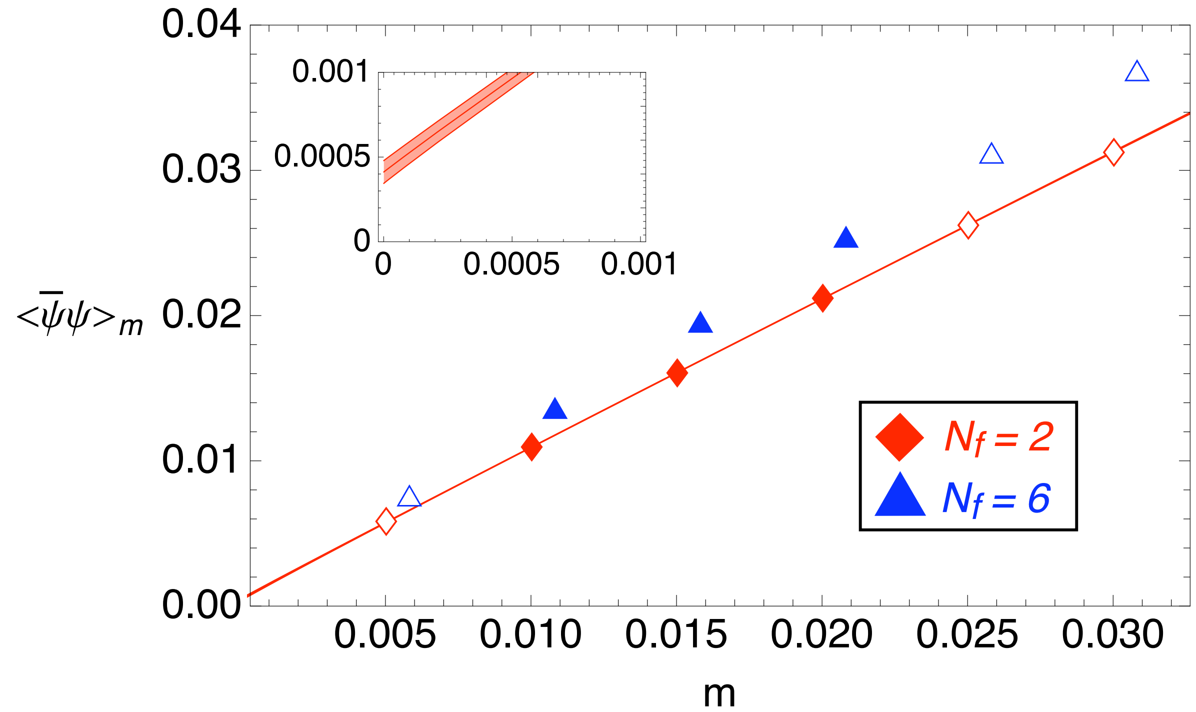
$$\langle\overline{\psi}\psi\rangle_m = F^2 B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_C - \frac{{N_f}^2 - 1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

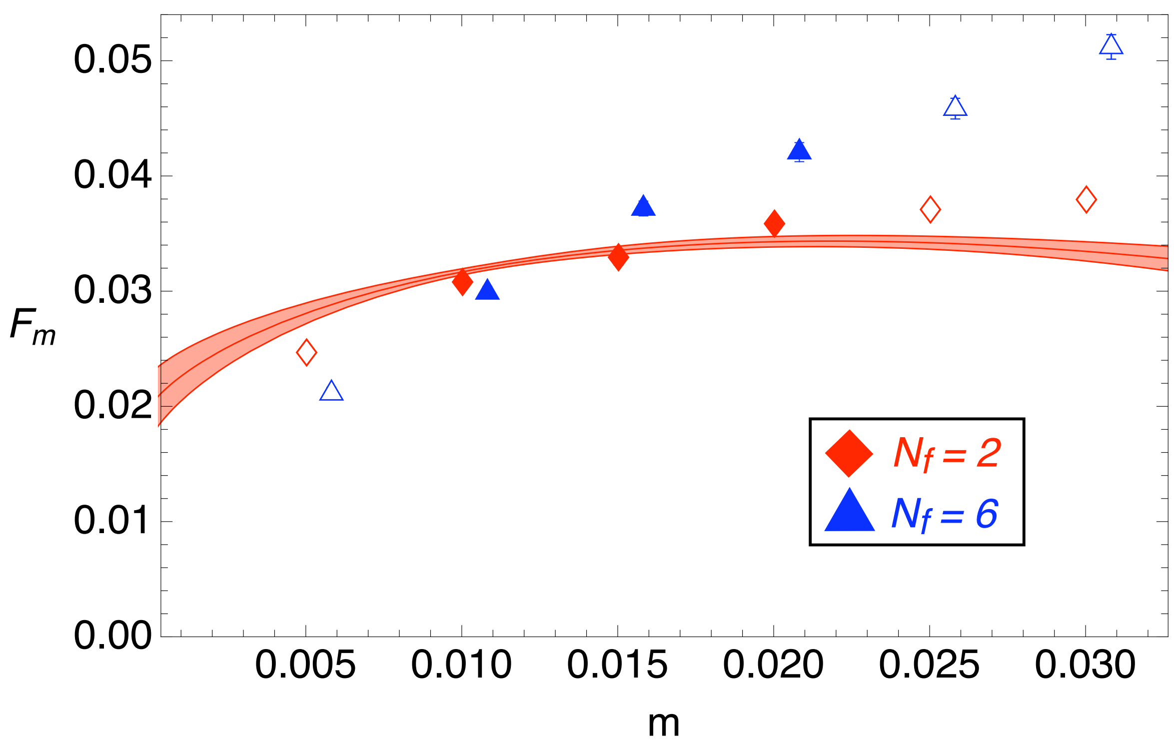
- NLO terms linear in N_f - lighter mass needed to fit with more fermions.
- Linear divergence in chiral condensate:

$$\alpha_c \supset m\Lambda^2 \sim m/a^2$$

Data and chiral fits

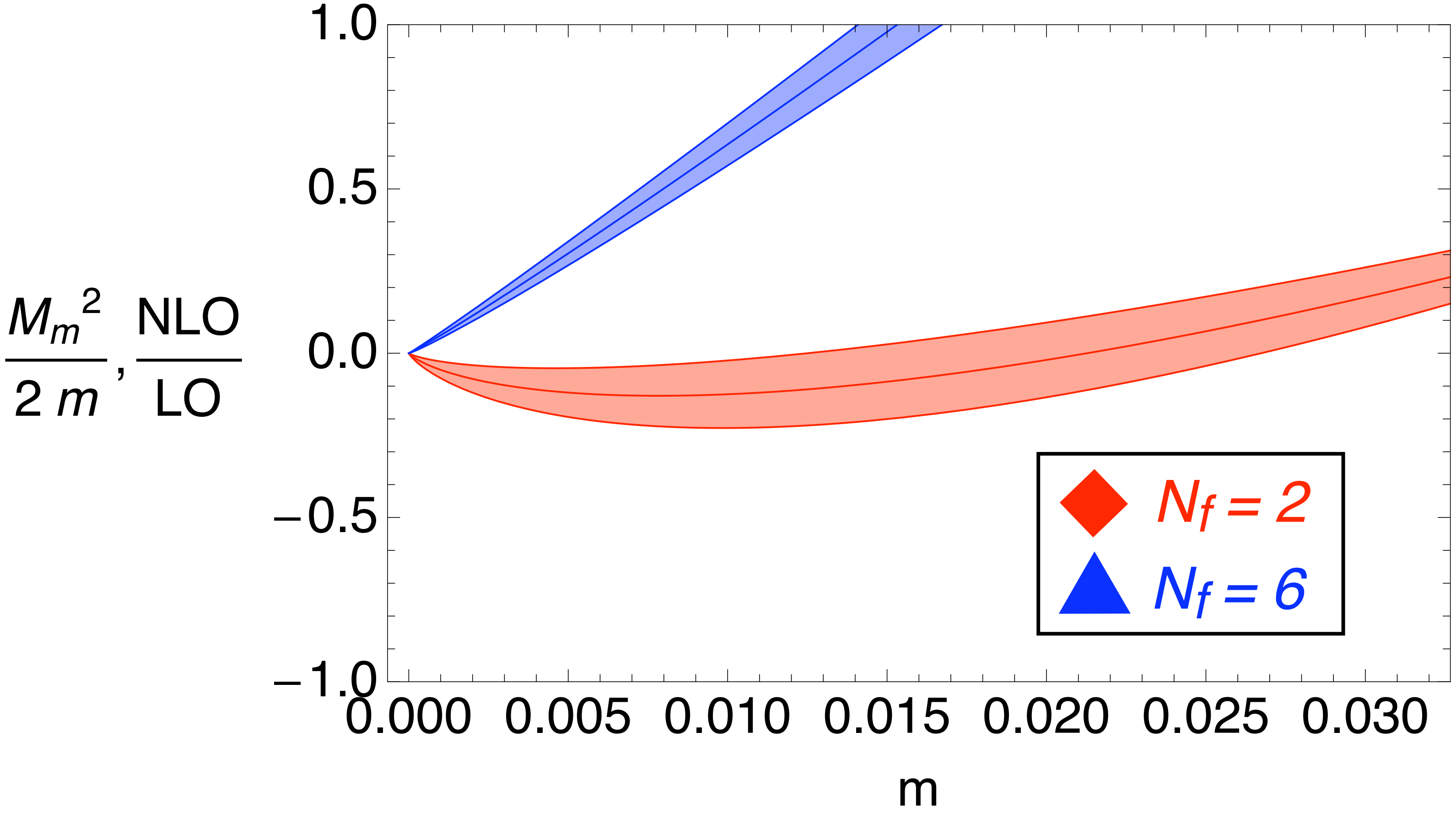






- Joint NLO chiral fit at N_f=2.
- Similar joint fit *fails* at N_f=6:
  - F_m NLO contributions > LO, by inspection!
  - Can fit pion mass and condensate without F_m, but predicted F much too high.
  - Lighter masses likely needed at 6 flavors

Data and chiral fits



<u>χPT expansion:</u>
consistent at Nf=2, poor at Nf=6 over mass range simulated.

<u>Sanity check: Nf=2 results vs. known values</u>

	LSD $N_f = 2$	known value
M_ρ / f_π	$10.9(1.6)$	$8.39(3)^a$
$\langle \overline{\psi}\psi \rangle / f_\pi^3$	$52.0(13.5)$	$36.2(6.5)^b$
$M_\rho r_0$	$0.494(28)$	$0.561(44)^c$

a) http://pdg.lbl.gov
b) M. Jamin, Phys. Lett. B538, 71 (2002) + renormalization
c) A. Gray et al., Phys. Rev. D72, 094507 (2005)

Ratios of ratios...

GMOR relation: $M_m^2 F_m^2 = 2m\langle\overline{\psi}\psi\rangle_m$ (leading order)

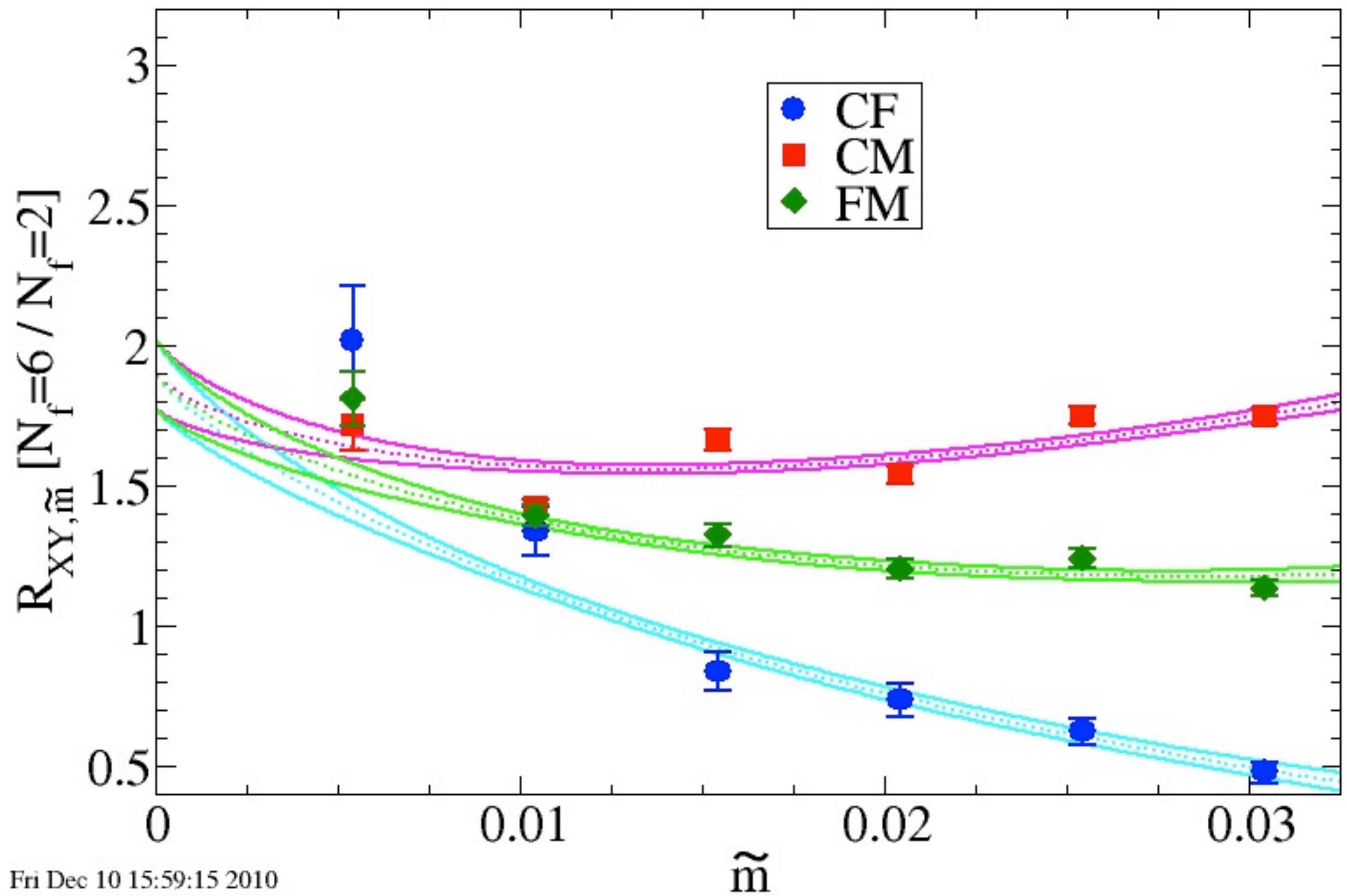
$$
\begin{array}{lcl}
(\text{CM}) & \dfrac{(M_m^2/2m)^{3/2}}{\langle\overline{\psi}\psi\rangle_m^{1/2}} & \searrow \\
(\text{FM}) & \dfrac{M_m^2}{2mF_m} & \xrightarrow{m \to 0} \quad \langle\overline{\psi}\psi\rangle/F^3 \\
(\text{CF}) & \dfrac{\langle\overline{\psi}\psi\rangle_m}{F_m^3} & \nearrow
\end{array}
$$

Compare two different theories: $R^{(N)} = \dfrac{[\langle\overline{\psi}\psi\rangle/F^3]_{N_f=N}}{[\langle\overline{\psi}\psi\rangle/F^3]_{N_f=2}}$

$$R^{(N)}_{XY,\tilde{m}} = R^{(N)}\left(1 + \alpha^{(N)}_{R,XY}\tilde{m} + \beta^{(N)}_{R,XY}\tilde{m}\log\tilde{m} + \ldots\right)$$

$(\tilde{m} = \sqrt{m_2 m_N})$

Condensate Enhancement



Lattice scheme: $R^{(6)} = 1.95(12)$

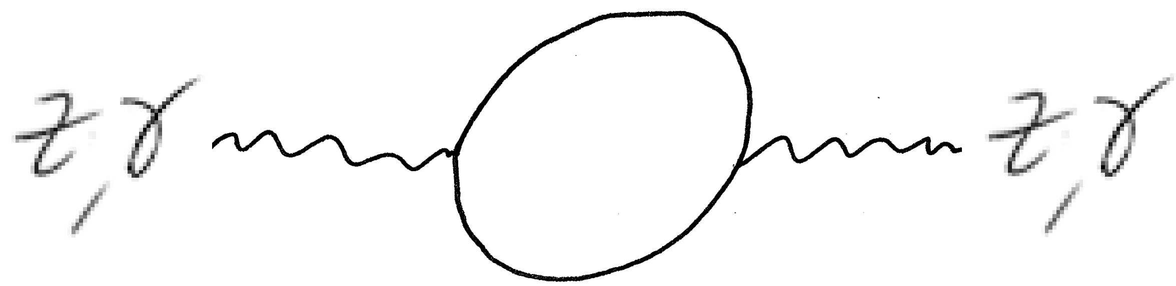
Renormalized: $R^{(6)}_{\overline{\mathrm{MS}}} = 1.60(10)$

Pert. theory est: $R^{(6)}_{\overline{\mathrm{MS}},pt} \lesssim 1.15$

(at 3.85 GeV!)

S-parameter

S is sensitive to electroweak "oblique corrections", i.e. vacuum polarization of EW gauge bosons at zero momentum transfer:



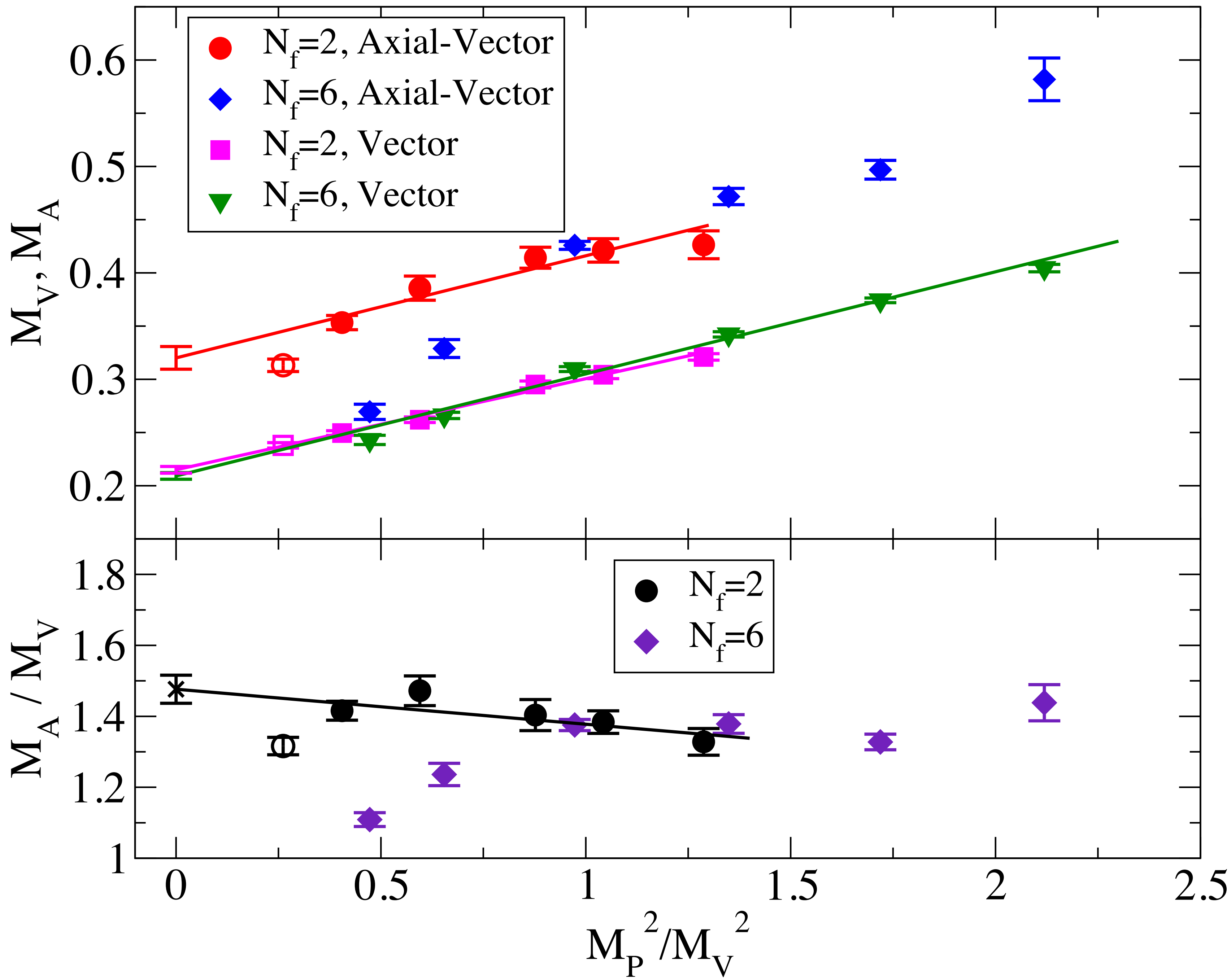
$$S = 16\pi(\Pi'_{33}(0) - \Pi'_{3Q}(0))$$

$$\Pi_{XY}(q^2) \equiv \frac{1}{d-1}\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)\int d^d x e^{iq\cdot x}\langle J_X^\mu(x) J_Y^\nu(0)\rangle$$

Can be re-expressed in terms of vector and axial-vector currents:

$$S = -4\pi(\Pi'_{VV}(0) - \Pi'_{AA}(0))$$

Parity Doubling (?)



Momentum dependence

- To extract the slope at zero momentum (and thus *S*), fit V-A correlators as functions of q^2, at fixed m and N_f.
- Operator-product expansion constrains the functional form at large momentum:

$$\Pi_{V-A}(q^2) \xrightarrow{q^2\to\infty} \frac{N_{TC}}{8\pi^2} m^2 + \frac{m\langle\overline{\psi}\psi\rangle}{q^2} + \mathcal{O}(\alpha) + \mathcal{O}(q^{-4})$$

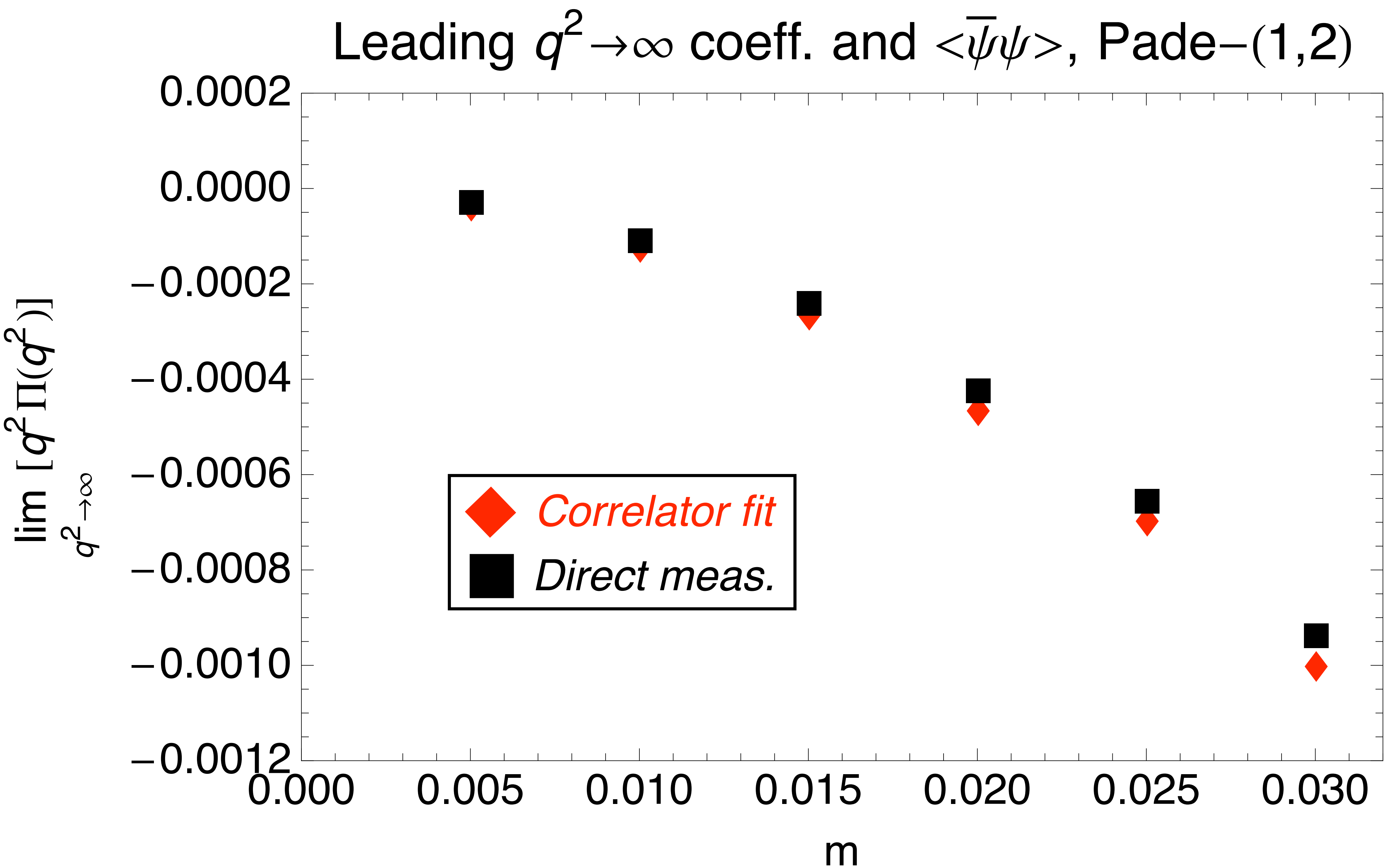
[M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B 147 (1979)]

Fit to Pade-(m,n) approximants:

$$\boxed{\Pi_{V-A}(q^2) = \frac{\sum_m a_m q^{2m}}{\sum_n b_n q^{2n}}}$$

Pade-(1,2) is found to yield good χ^2, stable results w/r/t fit range.

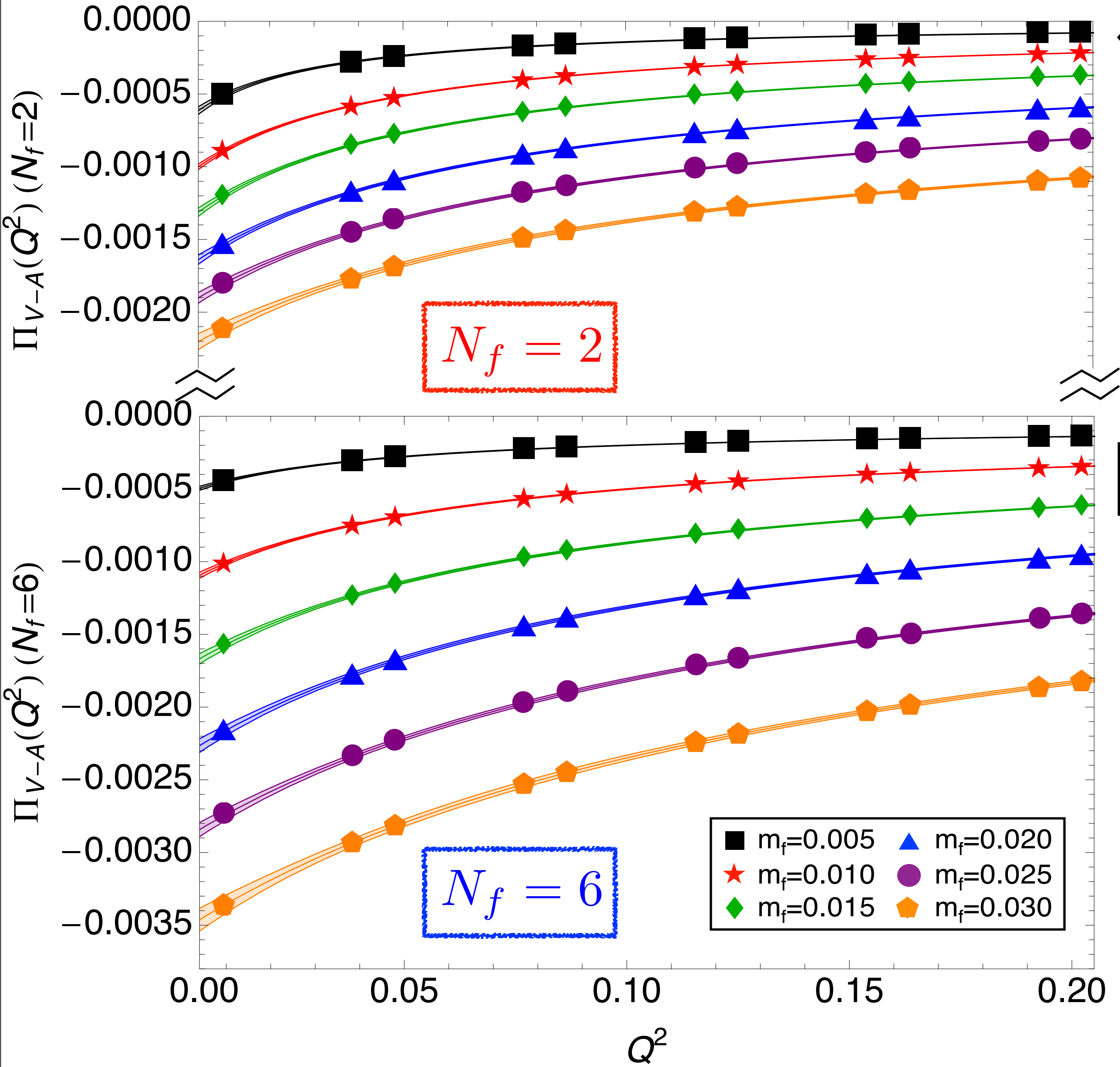
Momentum dependence



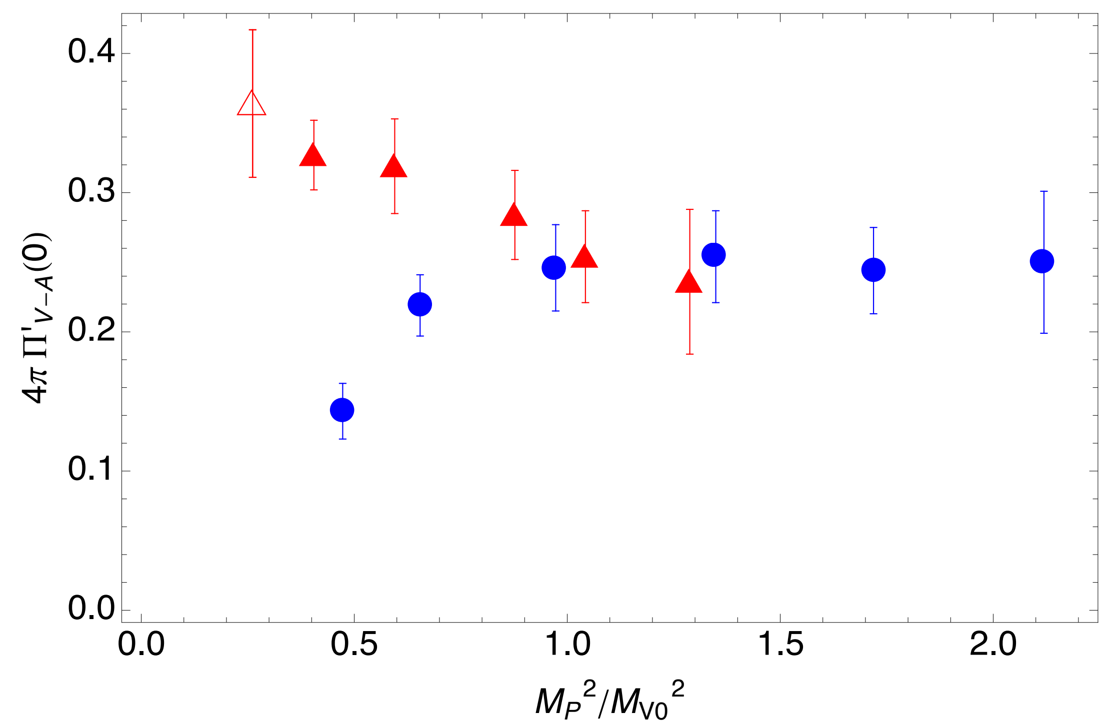
Excellent agreement between direct measurement and OPE

Fit results

(m=1, n=2)



$$\Pi_{V-A}(q^2) = \frac{\sum_m a_m q^{2m}}{\sum_n b_n q^{2n}}$$



Expect agreement in the quenched limit $M_P^2 \to \infty$

From slope to S

$\sim 4\pi\Pi'_{V-A}(0)$

$$S = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left\{(N_f/2)\left[R_V(s) - R_A(s)\right] - \frac{1}{4}\left[1 - \left(1 - \frac{m_h^2}{s}\right)^3 \Theta(s - m_h^2)\right]\right\}$$

ref. Higgs mass; we take $m_h \equiv M_{V0}$ (=1 TeV, roughly)

Standard model subtraction:

- Removes the contribution of standard model Higgs doublet to S
- IR divergent - cancels precisely with divergence in the spectrum!

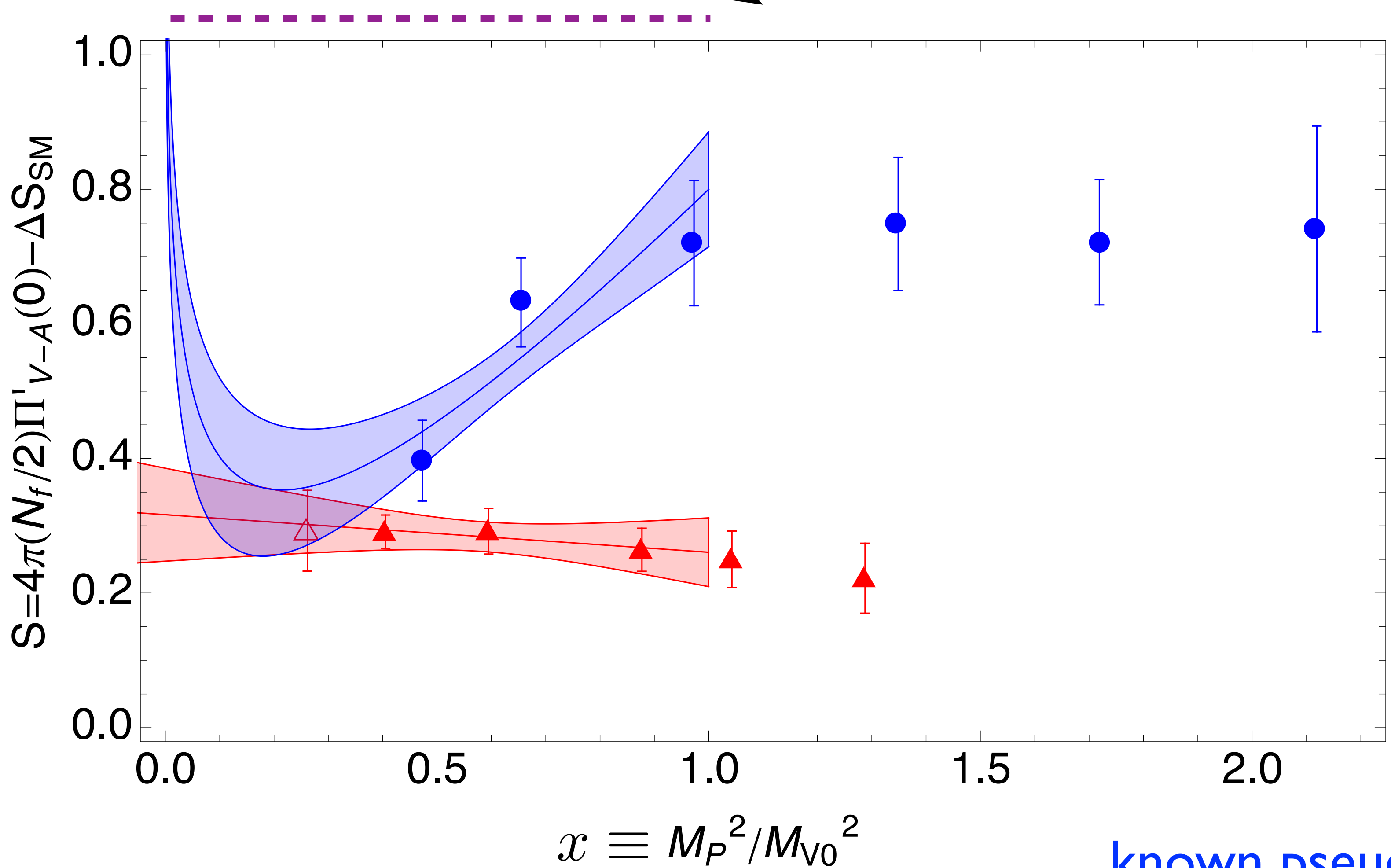
$(H, \phi \to \pi_T)$

Integrate: $$\Delta S_{SM} = \frac{1}{12\pi}\left[\frac{11}{6} + \log\left(\frac{M_{V0}^2}{4M_P^2}\right)\right]^*$$

$^*\left(\frac{M_{V0}^2}{M_P^2} < 1/4\right)$

From slope to S

“naive scaling” prediction $S_{N_f=6}/S_{N_f=2} = 3$



known pseudo-NGB contribution

Simple linear fit: $$S(x) = A + Bx + \frac{1}{12\pi}\left(\frac{N_f^2}{4} - 1\right)\log(1/x)$$

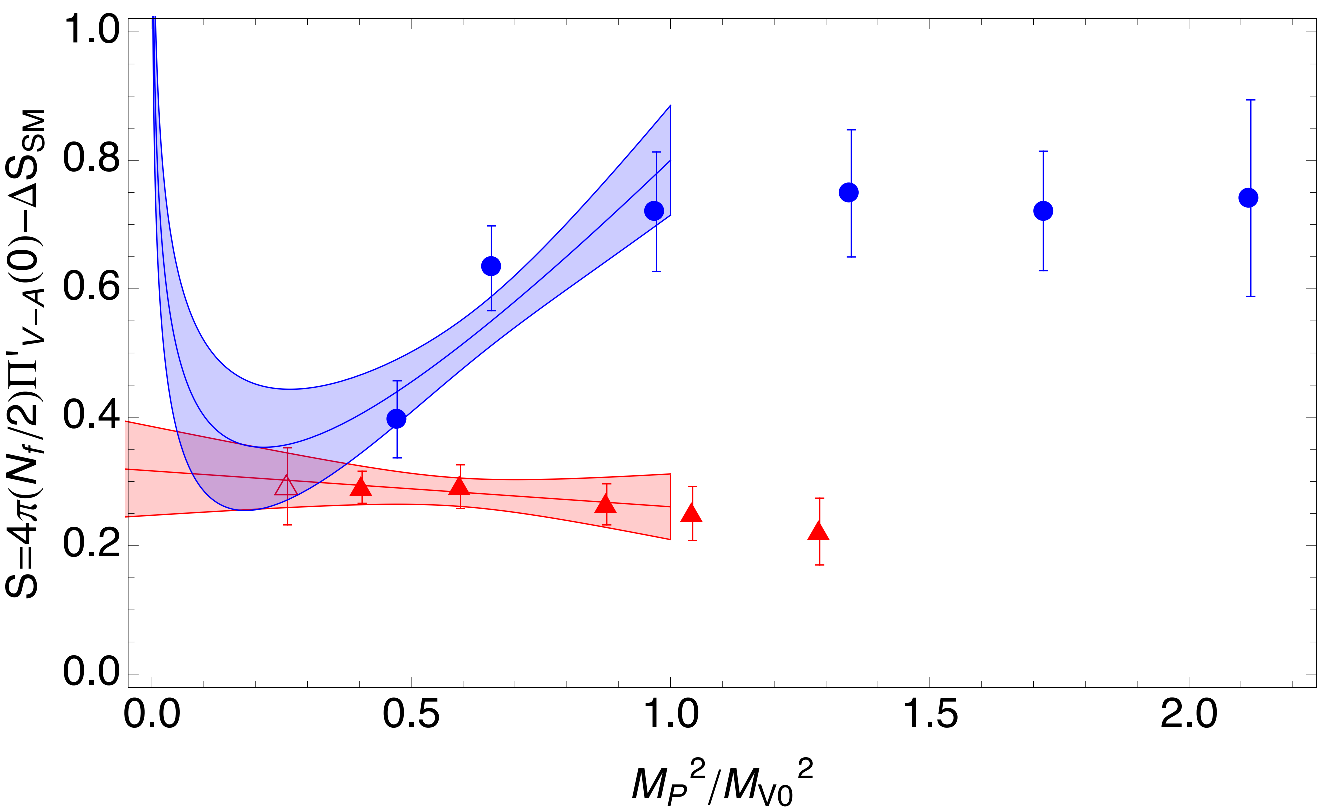
At two flavors, S(m=0) = 0.35(6) - consistent with other results

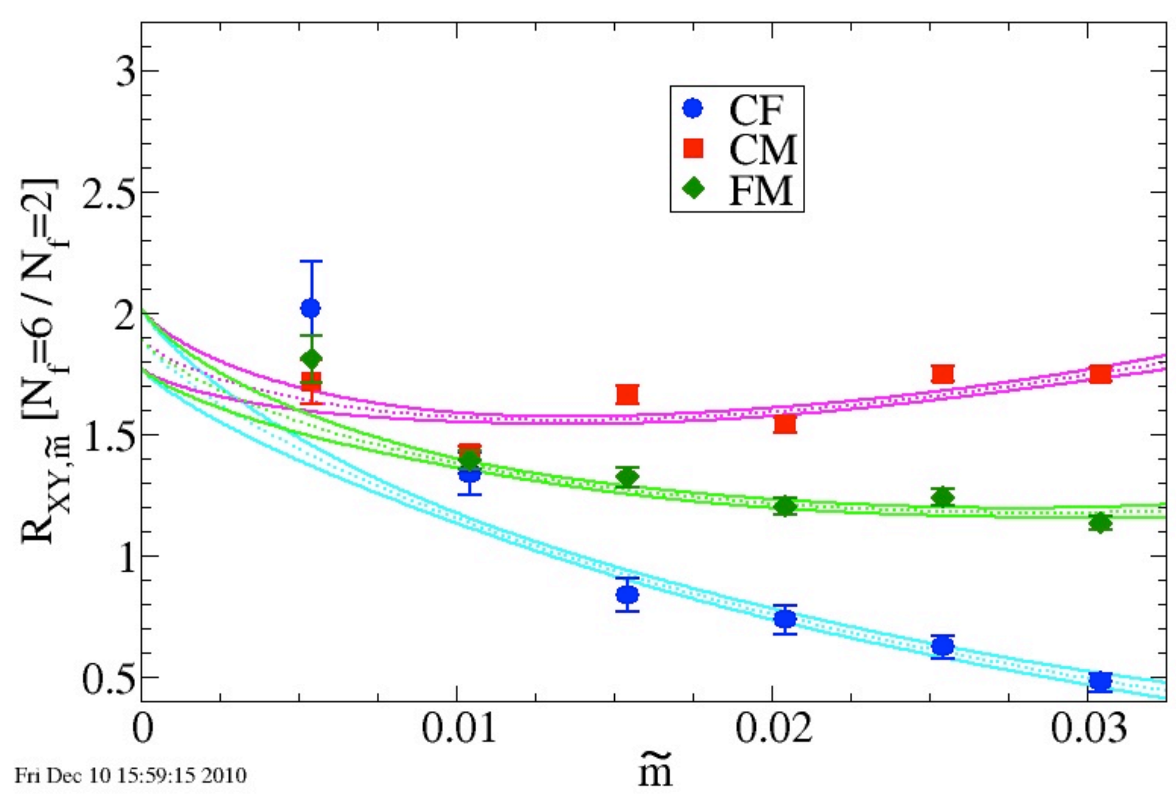
Next Steps

- Improving the results so far:
  - More detailed study of systematic effects, especially finite-volume corrections
  - Additional run at m=0.0075 in progress
- Simulation at N_f=10 (ongoing)
- Two-color gauge theories - code under development

Conclusion

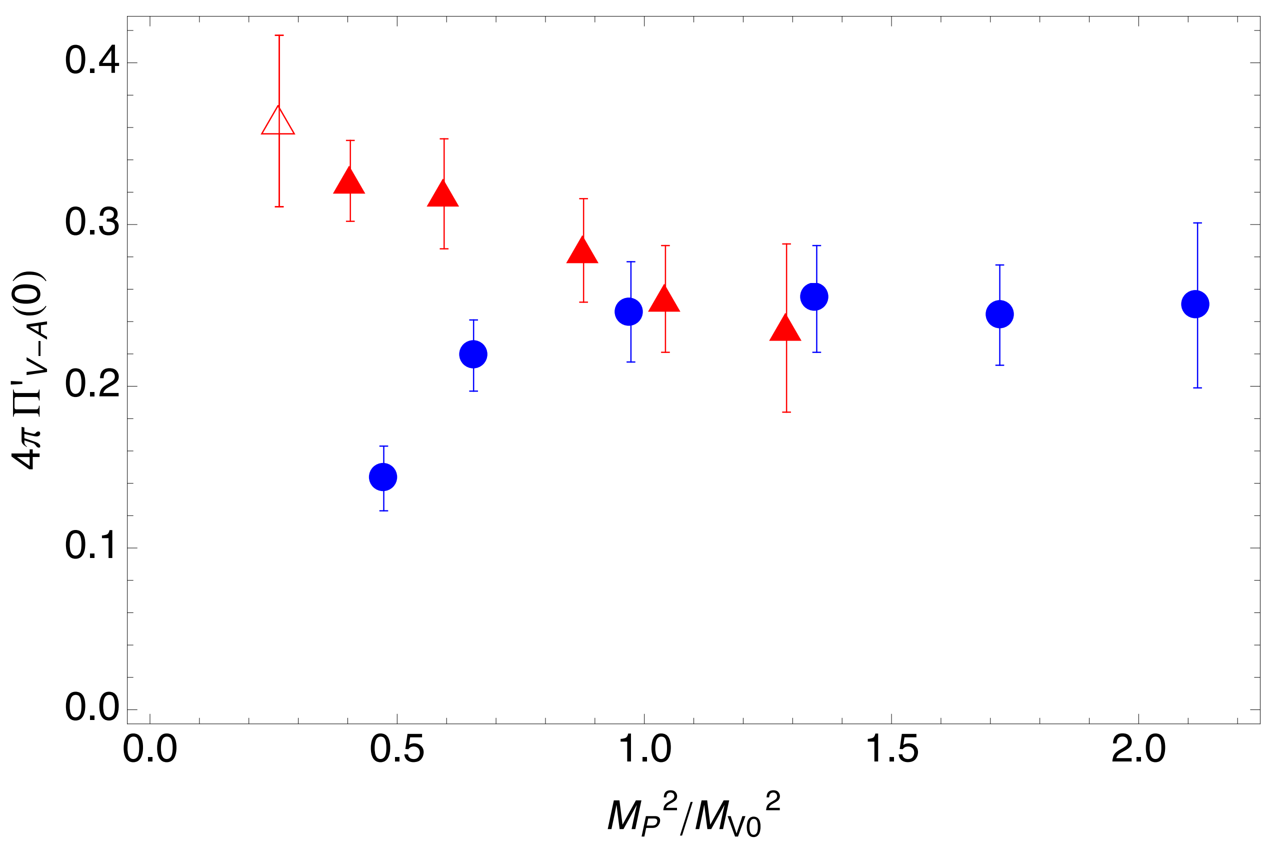
- Even far from the edge of the conformal window, strong indications of changing dynamics as Nf increases
- With scale-matched ensembles of gauge configurations generated, the hard part is done - lots of things to look at!

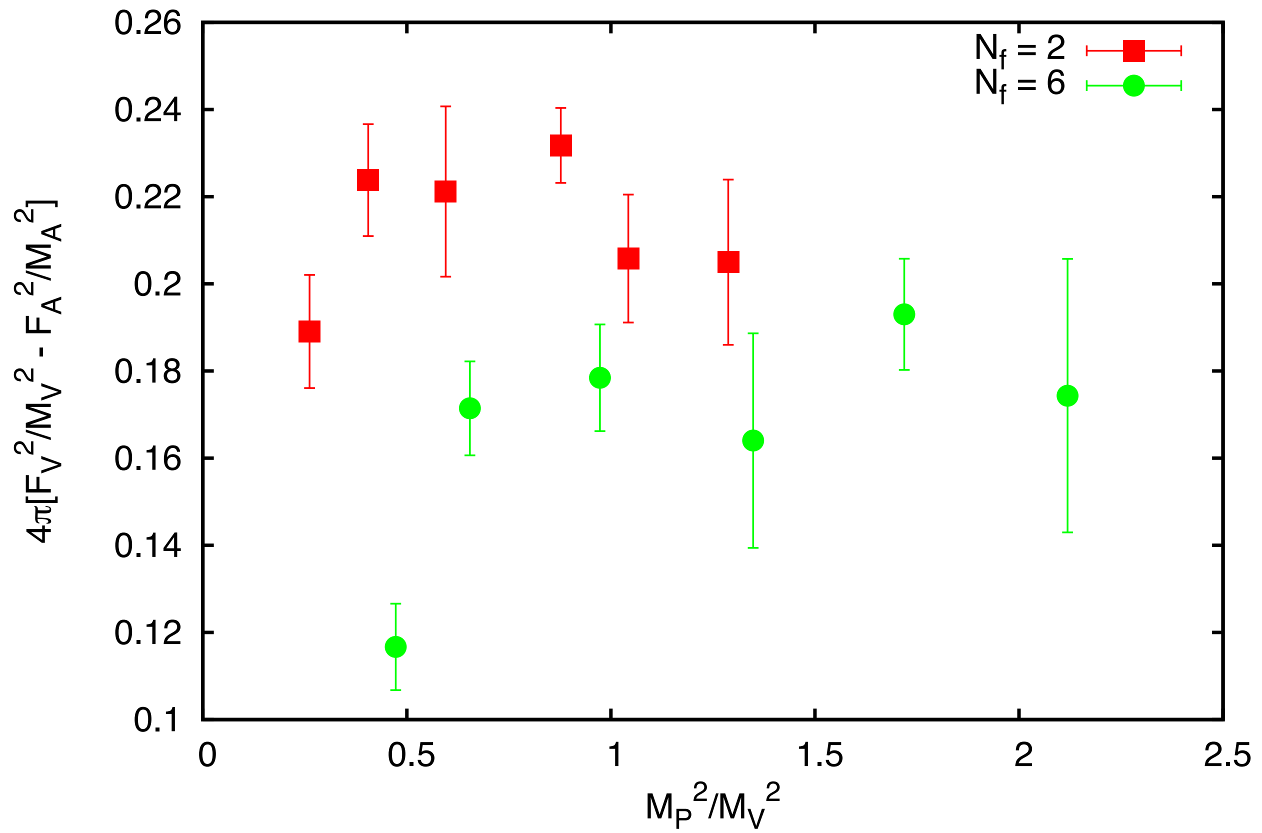




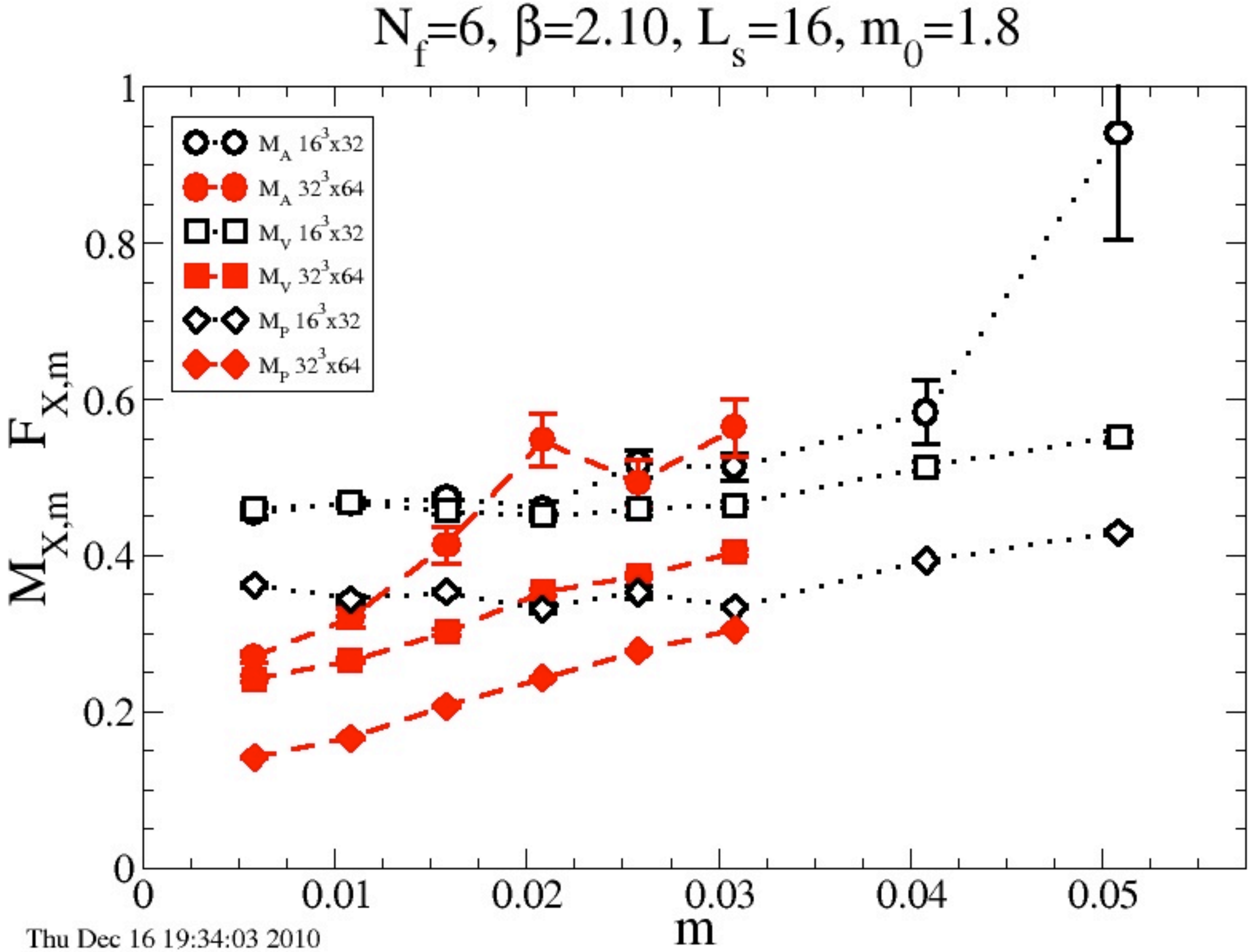
Backup Slides

S from pole dominance

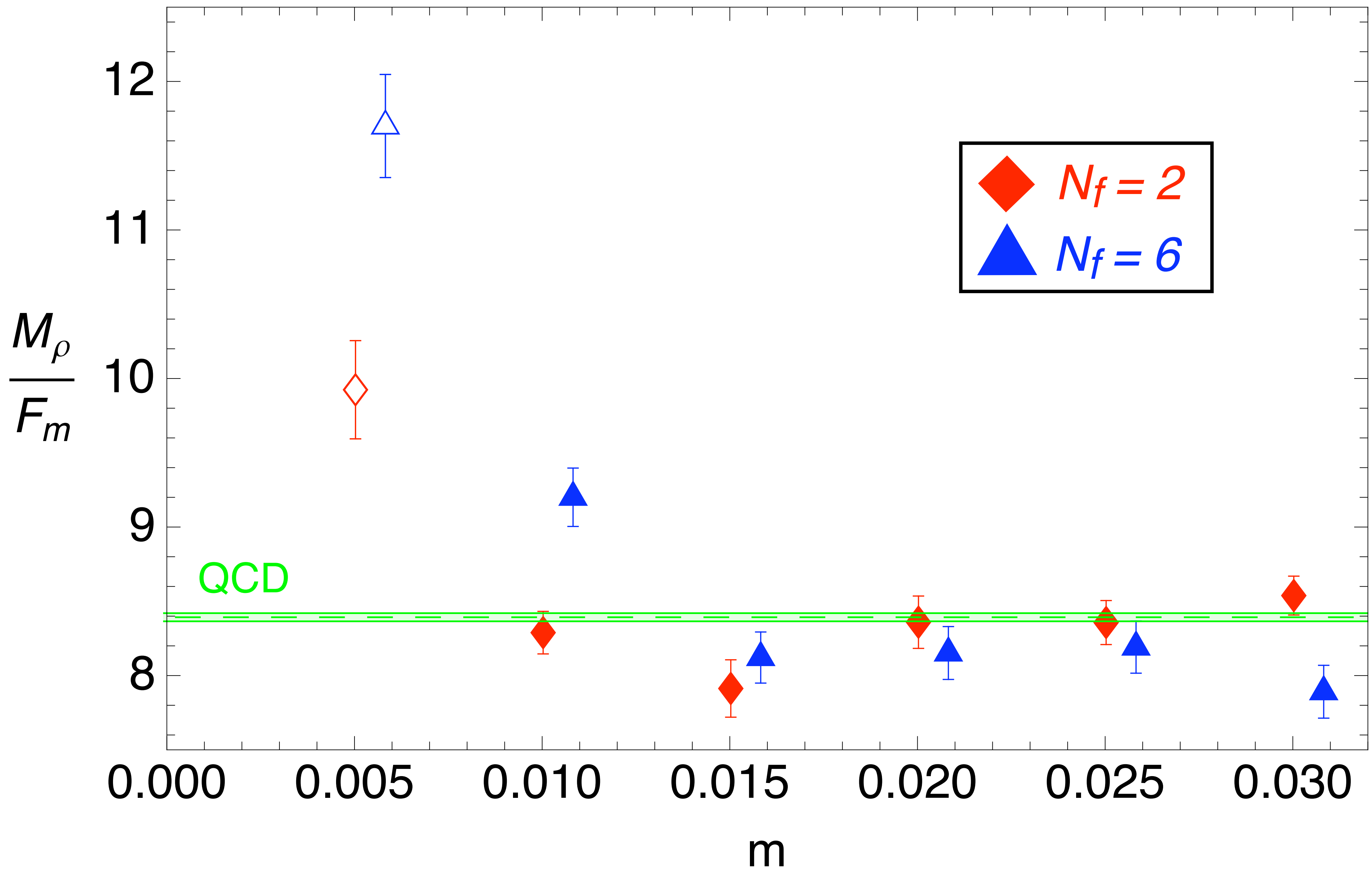




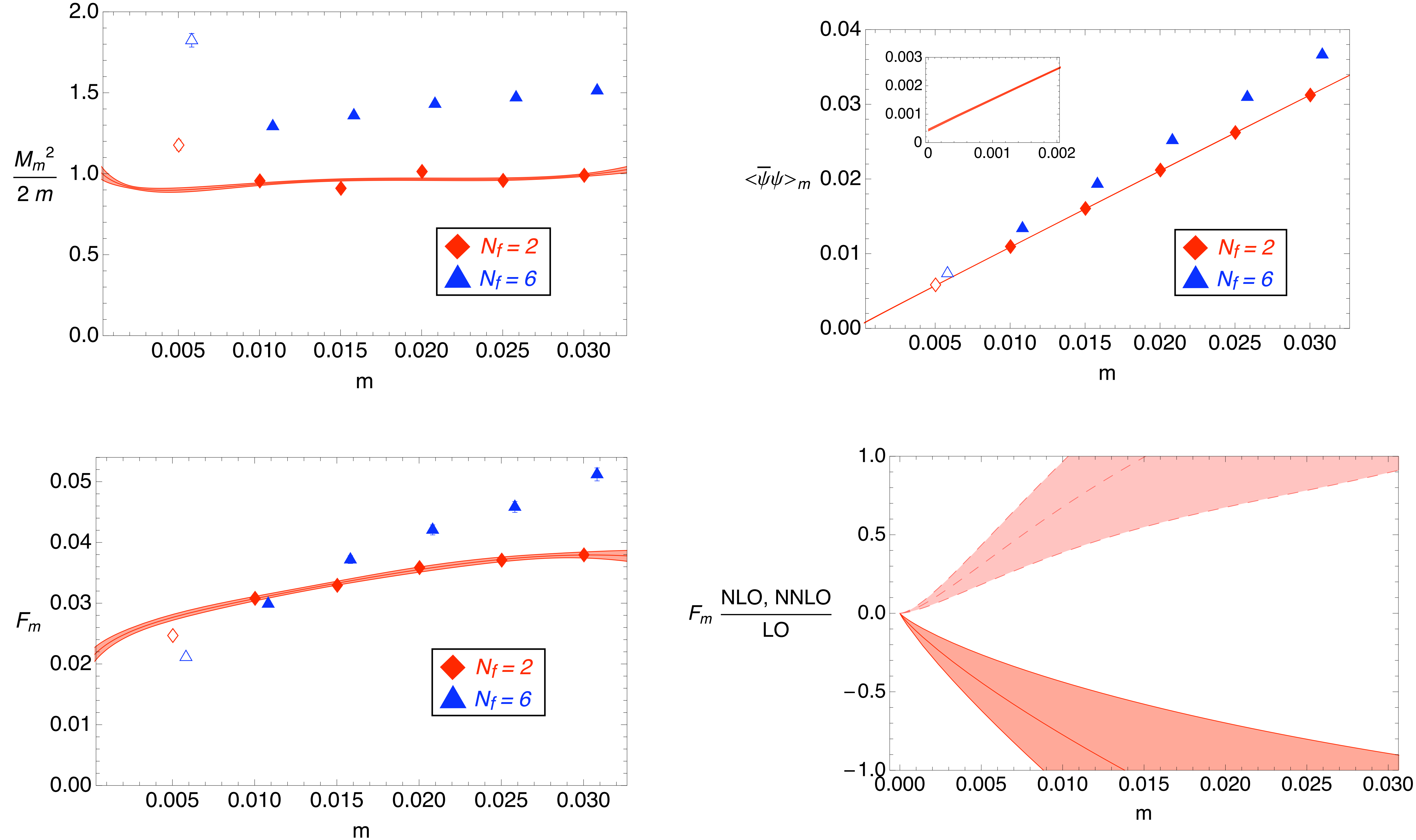
Finite-Volume Effects?



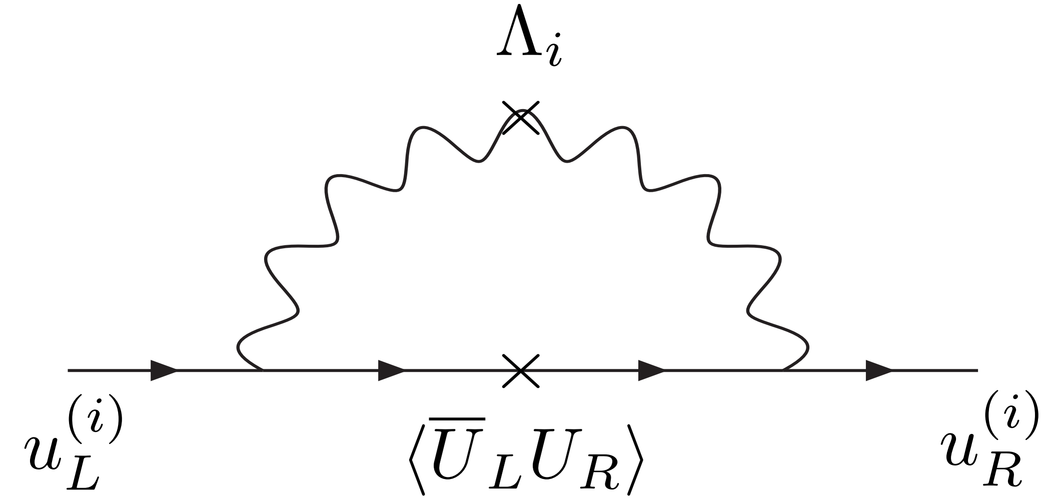
Fixing the right scale?



NNLO chiral fits



Mass generation and extended technicolor



$$m_q^{(i)} = \frac{8\pi\eta\Lambda_{TC}^3}{3\Lambda_i^2} \quad \Longrightarrow \quad \begin{cases} \Lambda_1 \simeq 300 \ \mathrm{TeV} \\ \Lambda_2 \simeq 15 \ \mathrm{TeV} \\ \Lambda_3 \simeq 1.3 \ \mathrm{TeV} \end{cases}$$

$$\Lambda_{TC} = 300 \ \mathrm{GeV}$$